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Ontological Aspects of Computing Analogies

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Introduction

In AI, there is an increasing interest in examining ontologies for a variety of applications. Classical ontologies can have different forms ranging from lattice-like structures (Ganter & Wille, 1996) to less restricted semantic networks (Peters & Shrobe, 2003).

Analogical reasoning has a long tradition in cognitive science and AI. The monograph Gentner, Holyoak & Kokinov (2001) is a good summary of recent theories for analogies. An important tool for modeling analogies is anti-unification (AU), introduced in Plotkin (1970). AU is a framework to compute generalizations of source and target which in turn can be used to establish an analogical relation (Schmid, Gust, Kühnberger & Burghardt, 2003). We will extend AU to so-called heuristic-driven theory projection (HDTP) to model analogical reasoning processes.

The Algorithm HDTP-A

We will present the algorithm HDTP-A computing generalizations together with their corresponding substitutions given a source and a target domain (Table 1). HDTP-A is based on an implementation in Gust, Kühnberger & Schmid (2003).

Table 1: The algorithm HDTP-A

Input: A set of axioms S of the source domain in a language L_S inducing a theory Th_S and a set of axioms T of the target domain in a language L_T inducing a theory Th_T.

Output: A generalized set of axioms G in a language $L^+_{S \oplus T}$ together with corresponding substitutions inducing a theory Th_G.

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$T$ = axioms of the target domain sorted by a heuristics $h$
$S$ = axioms of the source domain
$G$ = empty list of axioms of generalized theory
$\Theta_1 = \Theta_2$ = empty substitution
$Th_T^{A_h} = Th_T$
FOR $\phi \in T$
    $\phi = normal\_form(\phi)$
    SELECT $\psi \in S$
        $\psi = normal\_form(\psi)$
        IF not $same\_structure(\phi, \psi)$ REJECT
        SELECT $(\xi, \Theta_1, \Theta_2) \in anti\_instances(\phi, \psi, \Theta_1, \Theta_2)$
    WITH $\xi$ best according to a heuristics $h'$
    IF $h'(\xi) >$ a given threshold
        ADD $\xi$ to $G$
        ADD $\xi\Theta_2$ to $T_T^{A_h}$
        REMOVE $\psi$ from $S$
    ELSE FAIL
END FOR
FOR $\psi \in S$
    $\phi = transfer(\psi, \Theta_1, \Theta_2)$
    IF $T_T^{A_h} \vdash \neg\phi$ CONTINUE
    IF $oracle(\phi) = FALSE$ CONTINUE
    ADD $\phi$ to $Th_T^{A_h}$
    ADD $generalize(\psi, \Theta_1)$ to $G$

END FOR
```

The input is given by S (source) and T (target) inducing corresponding theories Th_S and Th_T. The output of the algorithm is a set of axioms (facts and laws) G inducing a theory Th_G generalizing source and target. The algorithm chooses an axiom from T governed by a heuristics h and searches an axiom from S to generalize both. Possible heuristics h that can be used for choosing axioms are *"Select simple axioms first"*, or *"Select axioms with a maximal number of shared terms w.r.t. already generalized terms"*. Additionally a heuristics h' is needed to select an anti-instance (AI) from all computed generalizations. Examples for such heuristics are *"Select an AI with minimal length of substitutions"* or *"Select an AI with a minimal number of second-order objects"*. After a successful generalization the resulting axiom is added to Th_G. This processes is recursively applied to all axioms in T. Finally remaining axioms in S can be transferred to T governed by the already computed substitutions as long as consistency of the extended theory and consistency w.r.t. to oberservables (checked by experiments) is guaranteed. Examples of such transfers are discussed in Schmid, Gust, Kühnberger & Burghardt (2003).

An Example: The Rutherford Analogy

In Table 2, representations of the source (left side) and the target (right side) of the Rutherford analogy are given. We consider *planet* and *sun* to be objects. Observable properties are *mass*, *distance*, and *force*. We assume certain laws that govern the behavior of objects in the solar system. Concerning the conceptualization of the atom (right side) we assume that objects *electron* and *nucleus* are given and observable properties are *electric charge*, *mass*, and *Coloumb force*. We presuppose that the *electron* and the *nucleus* have a *mass* and an *electric charge*. Finally we can perform an experiment (an abstract representation of the Rutherford experiment) to test whether analogical transfers yield valid results. Here are the computed generalizations:

Source Theory S	**Target Theory** T	**Generalized Theory** G
$mass(s) > mass(p)$	$mass(n) > mass(e)$	$mass(Y) > mass(X)$
$rev\_around(p,s)$	$rev\_around(e,n)$	$rev\_around(X,Y)$
$gravity(p,s,t) > 0$	$coloumb(e,n,t) > 0$	$F(X,Y,t) > 0$
$dist(p,s,t) > 0$	$dist(e,n,t) > 0$	$dist(X,Y,t) > 0$

Integrating Ontologies in HDTP-A

Ontologies typically order concepts hierarchically using a subsumption relation. Based on Stumme & Maedche

Table 2: Modeling the physics of the solar system and the atom model

types	*types*
real, object, time	*real, object, time*
entities	*entities*
planet : object ; sun : object	*electron : object ; nucleus : object*
functions	*functions*
observable mass: $object \times time \to real \times \{kg\}$	*observable mass:* $object \times time \to real \times \{kg\}$
observable dist: $object \times object \times time \to real \times \{m\}$	*observable dist:* $object \times object \times time \to real \times \{m\}$
observable gravity: $object \times object \times time \to real \times \{N\}$	*observable electric_charge:* $object \to real \times \{eV\}$
observable centrifugal: $object \times object \times time \to real \times \{N\}$	*observable coloumb:* $object \times object \times time \to real \times \{N\}$
facts	*facts*
revolves_around(planet, sun)	$mass(nucleus, t) > mass(electron, t)$
$mass(sun, t) > mass(planet, t)$	$electric\_charge(electron) < 0$
$\forall t : time : gravity(planet,sun,t) > 0$	$electric\_charge(nucleus) > 0$
$\forall t : time : dist(planet,sun,t) > 0$	$\forall t : time : coloumb(electron,nucleus,t) > 0$
laws	*experiment*
$\forall t : time, o_1 : object, o_2 : object :$	$\forall t : time : dist(electron,nucleus, t) > 0$
$dist(o_1, o_2, t) > 0 \wedge gravity(o_1, o_2, t) > 0 \quad \to$	
$\exists force : force(o_1, o_2, t) < 0 \wedge$	
$force(o_1, o_2, t) = centrifugal(o_1, o_2, t)$	
$\forall t : time, o_1 : object, o_2 : object :$	
$dist(o_1, o_2, t) > 0 \wedge \text{centrifugal}(o_1, o_2, t) < 0 \quad \to$	
$revolves\_around(o_1, o_2)$	

(2001), we define ontologies as follows.

Definition 1 *An ontology is a tuple* $\langle C, \texttt{is\_a}, R, \sigma \rangle$ *s.t.* C *is a set of concepts,* $\texttt{is\_a} \subseteq C \times C$ *is a partial order on* C*,* R *is a set of relations, and* σ *is an arity function.*[1]

The crucial feature in this definition is the subsumption relations `is_a`: Concepts specialize to subconcepts and generalize to superconcepts. In Figure 1, a corresponding hierarchical structure for the solar system of the example above is depicted. The idea is to use such ontologies to block generalizations that do not have a common superconcept. If the ontology has a tree-like structure like in Figure 1, then we need to specify subtrees that allow generalizations. In Figure 1 we would like to allow a generalization of *gravitation* and *centrifugal* and (under certain circumstances) the generalization of *force* and *distance.*

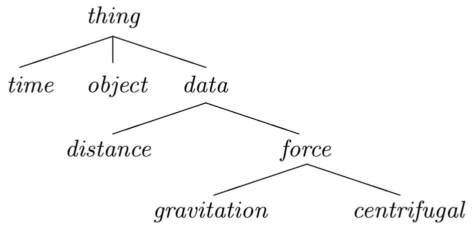

Figure 1: A possible ontology for for the model $\mathfrak{M}_1$.

Definition 2 *Assume an ontology* $O = \langle C, \texttt{is\_a}, R, \sigma \rangle$ *and a set* $C_{gen} \subseteq C$ *are given.*[2] *The generalizable expressions* O' *relative to a function Sort mapping terms to sorts are defined as follows:*

> *If* t_1 *and* t_2 *are terms, then* $\langle t_1, t_2 \rangle \in O'$ *iff there exists* $c \in C_{gen}$ *with* $Sort(t_1)$ `is_a` c *and* $Sort(t_2)$ `is_a` c*.*
>
> *If* ϕ *and* ψ *are complex, then* $\langle \phi, \psi \rangle \in O'$ *iff all corresponding subterms are generalizable.*

Using the definition of generalizable expressions the modified FOR-loop of the algorithm HDTP-A is depicted in Table 3 (given T and S, and an ontology O).

Table 3: The modified algorithm HDTP-A

```
...
FOR $\phi \in T$
    $\phi = normal\_form(\phi)$
    SELECT $\psi \in S$
        $\psi = normal\_form(\psi)$
        IF not $same\_structure(\phi, \psi)$ REJECT
        IF not $\langle \phi, \psi \rangle \in O'$ REJECT
        SELECT $(\xi, \Theta_1, \Theta_2) \in anti\_instances(\phi, \psi, \Theta_1, \Theta_2)$
    WITH $\xi$ best according to a heuristics $h'$
    IF $h'(\xi) >$ a given threshold
        ADD $\xi$ to $G$
        ADD $\xi\Theta_2$ to ${T_T^A}^h$
        REMOVE $\psi$ from $S$
    ELSE FAIL
END FOR
...
```

A test relative to a given ontology O is integrated in the algorithm HDTP-A in order to reduce the search space for possible generalizations. The corresponding ontological commitments are clearly domain dependent.

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[1] In this paper, R and σ are not relevant.

[2] The intended interpretation of C_{gen} is a set of leaves of an upper structure, in our example the set {*time, object, date*}.