What Can (and Can't) Make Problem Solving by Insight Possible? A Complexity-Theoretic Investigation

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Keywords: Problem Solving; Insight; Representation Restructuring; (Parameterized) Computational Complexity

Problem Solving by Insight

Much of human problem solving can be accounted for by Newell and Simon's classic state-search model, in which a representation of a problem is chosen based on previous experience and search is then performed within the space of problem-states associated with this representation until a state is encountered that satisfies the problem's goals, *i.e.*, a solution. This assumes that the representation chosen is correct, in that there are solutions that can be reached by search within that representation's problem-state space. If this is not so, insights are necessary to modify the initial representation such that search can succeed (Duncker, 1945).

Problem solving by insight can be construed as cycles of search alternating with applications of special representation restructuring operators until a solution is reached. Within the most formally-stated such theory, the Representation Change Theory (RCT) of Knoblich et al. (1999), a problem representation additionally consists of a set of constraints encoding both restrictions on the search process and the characteristics of those problem-states that are solutions. The entities comprising a problem-state are grouped into chunks, where each chunk corresponds to a pattern that has proven useful in previous instances of problem solving. At any given time, only one set of chunks (whose members may not be nested) is considered active. RCT proposes two representation restructuring operations, namely, the removal of a particular constraint (Constraint Relaxation) or the replacement of an active chunk by its immediately-nested chunks (De-Chunking). The classical application by Knoblich et al. of RCT to matchstick arithmetic problems is shown in Figure 1.

Problem solving by insight is widely viewed as being more difficult than search-based problem solving (Chu & MacGregor, 2010). Within RCT, it has been conjectured that problems whose solutions violate smaller numbers of constraints (Knoblich et al., 1999, p. 1535) or with fewer constraints in total (MacGregor & Cunningham, 2009, p. 133) should be easier to solve. Though consistent with empirical observations, the question remains whether such explanations are complete, in the sense that the increases in solution-frequency and/or speed are due to these factors by themselves or by these factors in collaboration with other so-far unnoticed limitations. In this poster, we investigate these claims using the methodology for analyzing computational-level models of cognitive theories described in van Rooij & Wareham (2008).

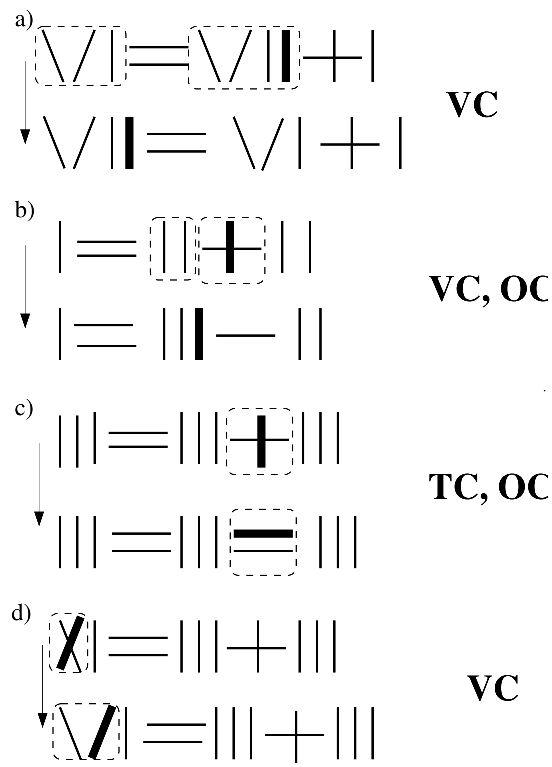

Figure 1: Solving Matchstick Arithmetic Problems within Representation Change Theory. In each problem, a single matchstick must be moved to make a mathematical formula correct. De-chunked chunks are denoted by dashed boxes. To the right of each problem are the constraints that were relaxed (VC: Value; OC: Operator; TC: Tautology).

Computational-level Model

A problem representation consists of a collection of entities and their relationships, a collection of chunks imposed on this collection, and a subset of those chunks comprising the currently active chunks. We model entity-relationship collections as predicate-structures, chunks as sub-predicate-structures, *i.e.*, a subset of the objects in a predicate-structure and all relationships in that structure that are based on the objects in this subset, and active chunks as non-nested collections of chunks that cover all objects (though not necessarily all predicates) in a predicate-structure. Given this, search operators are rules of the form $X \rightarrow Y$ that operate on predicate-structures, constraints are logic formulas which operate over

Table 1: Overview of parameters considered.

Name	Description
$\|k_C\|$	Maximum # of constraint relaxations
$\|k_D\|$	Maximum # of de-chunkings
$\|k_S\|$	Maximum # of search operator applications
$\|T\|$	Number of available chunk-types
$\|C\|$	Number of constraints
$\|O\|$	Number of available search operators
a	Maximum # of search-operator opportunities
$\|T_A\|$	Maximum # of active chunks

the objects, predicates, and chunks in a problem representation. and the constraint relaxation and de-chunking operators are the deletion of a constraint and the replacement of an active chunk c by one or more non-overlapping chunks that are nested inside and cover all objects in c, respectively. This yields the following input-output mapping:

PROBLEM SOLVING BY RCT-INSIGHT (PSRI)
Input: Chunk-type set T, search-operator set O, problem representation p with active chunk-set D, constraint-set C, and integers k_C, k_D, and k_S.
Output: A solution s for p that is derived by applying $\leq k_C$ constraint relaxation and k_D de-chunking operators followed by $\leq k_S$ search operators from O, if such an s exists, and special symbol $\perp$ otherwise.

Complexity Results and Discussion

Following convention in Computer (Garey & Johnson, 1979) and Cognitive (van Rooij, 2008) Science, we consider a cognitive theory tractable if its associated input-output mapping can be computed in polynomial time, *i.e.*, computed by an algorithm that runs in time upper-bounded by n^c where n is the input size and c is a constant.

Theorem 1 *PSRI is NP-hard when either $k_C = 0$ or $k_D = 0$.*

Theorem 1 establishes that, modulo the widely-believed conjecture that $P \neq NP$ (Fortnow, 2009), insight problem solving under RCT cannot be done in polynomial time. Note that this holds whether re-structuring consists purely of constraint relaxation or de-chunking, which implies that the focus to date on restricting only the amount of constraint relaxation to ease the difficulty of solving insight problems is in error.

This result also means that strong restrictions must be assumed to apply to the input domain of PSRI for RCT to be able to explain solution of insight problems by human beings. Let us formulate such restrictions in terms of the values of selected parameters, which are aspects of problem inputs. We say that a set K of one or more parameters renders an input-output mapping Π **fixed-parameter (fp)) tractable** if there is an algorithm for Π that runs in time upper-bounded by $f(K)n^c$, where f is an arbitrary function (Downey & Fellows, 1999; van Rooij, 2008). To investigate which restrictions suffice for rendering PSRI tractable, we performed fp-tractability analyses relative to the parameters in Table 1.

Theorem 2 *PSRI is not fp-tractable for $\{k_C, k_D, k_S, |T|, |C|, |O|, a\}$,*

Theorem 3 *PSRI is fp-tractable for $\{k_D, k_S, |C|, a, |T_A|\}$,*

Theorem 2 establishes that PSRI cannot be made easy even if the parameters in both published conjectures (k_C and $|C|$) are bounded simultaneously with a number of other plausible parameters. Theorem 3 provides the first provably complete explanation of the tractability of PSRI. This explanation is not totally satisfactory because (1) there is no empirical evidence that $|C|$ and a are small in practice and (2) the invoked parameter-set is not provably minimal, as it is possible that restricting some subset of these parameters in combination with $|T_A|$ may give fp-tractability. That being said, whether or not either of these objections are substantive can be settled by further experimental and theoretical research. Future research should also investigate whether additional parameters not considered here yield alternate complete explanations of the precise circumstances under which problem solving by insight is and is not possible, both under RCT and other proposed theories of restructuring-assisted problem solving (see Ash et al. (2009) and references).

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