Modeling Risky Decision Making by Cellular Automata

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Introduction

This paper proposes a new approach to the problem of decision making under risk (and partly under uncertainty) by a simple cellular automaton which can simulates well-known anomalies of risky choice. This field has been studied mainly by psychologists, economists, management scientists, and more recently neuroscientists so far. Various types of anomalous choice patterns which violate the prediction according to the probability theory and the expected utility theory are called paradoxes, or anomalies. Specifically, descriptive theory of risky choice should model the effects of event-splitting (and event-merging) to violate the first-order stochastic dominance, as well as the ambiguity aversion and the common consequence effect.

By editing branches and matching of rows in two gambles, one could invite anomalous choice which reverses mathematically the same choice, therefore decision maker's risk cognition permit a manipulation by the problem representation. Figure 1 and Figure 2 show examples (They are the choice problems 2 and 3.2 in Birnbaum (2008)).

Gamble A	Gamble B
90% win \$96	85% win \$96
5% win \$14	5% win \$90
5% win \$12	10% win \$12

Figure 1: Choice problem between A and B.

Gamble C	Gamble D
85% win \$96	85% win \$96
5% win \$96	5% win \$90
5% win \$14	5% win \$12
5% win \$12	5% win \$12

Figure 2: Choice problem between C and D.

Note that Gamble A is the same as Gamble C if two of the best branches (85% and 5% of wining \$96) are combined. Conversely, an event-splitting for the first branch of Gamble C generates 4-branch Gamble A. Similarly, Gamble B is made by an event-splitting for the last branch (the worst) of Gamble D. Gamble D is first-order stochastically dominated by Gamble C (i.e., the graph of C's cumulative probability is always not above than of D and strictly below at some state). However, many of human subjects choose A as reported in the experimentation study (The choice ratio are 73% and 6% for B and D).

Violation of stochastic dominance is a type of anomaly which cannot be explained by either Expected Utility Theory (EUT) or Cumulative Prospect Theory (CPT). Recently, Birnbaum's Transfer of Attention Exchange (TAX) theory is rediscovered to explain the event-splitting effect as well as other new paradoxes.

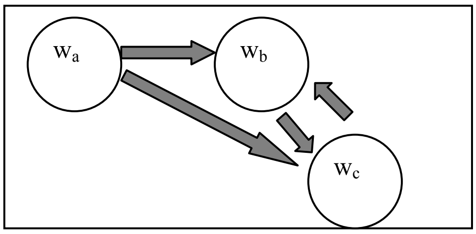

Figure 3: A three-branch TAX theory (Birnbaum (2008), p.471). Assume a DM who prefers 'a' to 'b' to 'c' and the probability weight $W(p) = p^{0.7}$ with 25% transfer along each arrow. Then, this model can predict a reversal in comparing between Gamble E = (\$100, 80%; \$50, 15%) and Gamble F = (\$100, 95%; \$7, 5%) by a simultaneous split of the probabilities of \$50 in E and of \$100 in F into 10% and 5%.

Two questions could have been pointed out. First, the TAX model proposed in order to predict event-splitting effects has the maximal reversal effect at simultaneous bisections for the best branch of originally inferior gamble and the worst branch of superior one. This may generate also a counterintuitive example (Namely, Gamble E' = (\$100, 80%; \$50, 7.5%; \$50, 7.5%) and Gamble F' = (\$100, 47.5%; \$100, 47.5%; \$7, 5%). Second, the theory lacks a consistent explanation of modularity-switching for any probability weight function. In order to model ambiguity aversion should be super-modular, while the function of event-splitting effect should be sub-modular.

In this paper, an alternative approach to modeling cognitive dynamics of risky choice by using cellular automata is proposed, and computational experiments which generate the anomalies of risky choices are demonstrated.

Cellular Automaton Model

Cellular Automaton

A cellular automaton is a collection of cells live in a grid (or torus) normally a Cartesian of some number of lines (or loops) each of which consists of a finite number of cells. Each cell has its own permissible state from a given set of numbers (or colors). Transition of states for each cell is governed by a given set of rules which depend on the state profiles of the neighbors.

The state of a cell changes over time according to the rule which may depend on the current state, and possibly on the previous states, of neighboring cells. The computer simulation of cellular automata mimics such processes iteratively. The shape as a whole is often complex even if the governing rules are simple, however, ordered patterns maybe emerged unpredictably. It is known that cellular automata have a capability of producing the universal Turing machine. (For more detail see Wolfram (2002).)

Stochastic Local q-Majority Vote

In order to model cognitive dynamics of risky choice and to simulates anomalies mentioned in the preceding section, we will devise the following two-dimensional cellular automaton which represents working memory of the decision maker.

- The universe U is a two-dimensional torus with the index modulo 10, consisting of cells c(X, Y), where X, Y = 1, 2, …, K. We assume K = 10 for the sake of practical computation.
- The neighborhood over which cells affect one another is a square which consists of nine cells including the center cell (i.e., the Moore neighborhood).
- A possible state of each cell at each step is assumed to be a triple of bits, i.e., eight is the number of permissible states for each cell at any time. Note that the three-bit pattern can represent any (in)transitive ordering of three objects, for example, a 110 stands for $x > y > z$. The first bit is a value 1 if x is preferred to y by the DM, else 0. The second and the third bits index the paired comparisons of y to z and of z to x respectively.
- A stochastic quotient Local Majority Vote is assumed to be *the rule of influence* which governs behavior of each cell affected from its neighborhood as the set of voters of the local polls for each step. The number q is assumed to be 2/9 (or 3/9) in the following computer simulation experiments.

The intuitive interpretation of cells are linearly matched two gambles each of which is decomposed into a hundred of "one-percent" branches. The first assumption reflects the key factor in this model is the sequences and spatial configuration of the matching two gambles --- a metaphor of battle on sphere which to decide whether gambles wins.

Local Majority Vote was first argued by Peleg (2000) in the context of algorithm in distributed computer network. It also seen as a version of the ACM (Axelrod, 1997) modified the neighbor similarity criteria by q-local majority vote. The stochastic q-version of LVM using the above assumptions produces various tiled patterns if q = 3/9 (Indo, 2012).

In each step this procedure randomly selects a neighbor (the focal member) of each cell (the center of neighborhood) and set polls for each bit by a q-majority vote in the neighborhood. That is, for each bit the center cell should adopt the focal member's evaluation if the counted number of neighbors who have the same bit value as the focal member exceeds $9q$.

Simulation Results

This section summarizes computer experimentations of the cellar automata described in the previous section which can simulate the anomalies of risky choice mentioned in the first section by using *artisoc* a toolkit of agent-based modeling.

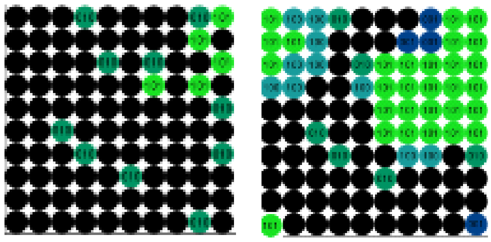

Figure 4: A cellular automaton of gamble comparison.

Figure 4 shows a cellular automaton which evaluates the two-branch gamble comparison described in Figure 3. In the left side of Figure 4 is the initial state of torus. A small colony of five Cell 101s represents five times of a match of two unit branches ($50, 1%) from Gamble E and ($7, 1%) from Gamble F. In the right side of Figure 4 depicts a tentative state where the Cell 101 colony is evolving to replace "black" circles each of which is Cell 000 a tie of ($100, 1%). The count of first bit indicates that E wins as a whole, and the remaining two bids record difference. Note that the ten Cell 010s, each of which stands for a match ($50, 1%) from E v.s. ($100, 1%) from F, are dispersed in the ground of eighty five Cell 000s. For many event-splitting that make unmatched event branches such like as E'-F' have similar configuration. However, a crowd rearrange of the Cell 010s could make F a final choice. For more detail see the poster and handout.

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References

Axelrod, R. (1997): The Dissemination of Culture: A Model with Local Convergence and Global Polarization. *Journal of Conflict Resolution*, *41* (2), 203-226.

Birnbaum, M. H. (2008). New paradoxes of risky decision making. *Psychological Review*, *115*(2), 463-501.

Peleg, D. (1996). Local Majority Voting, Small Coalitions and Controlling Monopolies in Graphs: A Review, In N. Santoro & P. Spirakis (Eds.), *Structure, Information and Communication Complexity: 3rd Colloquium, SIROCCO '96: Proceedings* (pp.152-189). Carleton University Press.

Indo, K. (2012). A Multi-Agent Simulation of Preference Formation by Local Majority Vote, mimeo.

Wolfram, S. (2002). *A New Kind of Science*. Champaign, IL: Wolfram Media.