

## A Computational Holographic Model of Memory for Abstract Associations

Matthew A. Kelly (Matthew.Kelly2@Carleton.ca)

Robert L. West (Robert.West@Carleton.ca)

Institute of Cognitive Science, Carleton University  
1125 Colonel By Drive, Ottawa, Ontario, K1S 5B6 Canada

### Abstract

How do humans learn the syntax and semantics of words from language experience? How does the mind discover abstract relationships between concepts? Computational models of distributional semantics can analyze a corpus to derive representations of word meanings in terms of each word's relationship to all other words in the corpus. While these models are sensitive to topic (e.g., tiger and stripes) and synonymy (e.g., soar and fly), the models have limited sensitivity to part of speech (e.g., book and shirt are both nouns). By augmenting a holographic model of semantic memory with additional layers of representations, we demonstrate that sensitivity to syntax relies on exploiting higher-order associations between words. Our hierarchical holographic memory model bridges the gap between models of distributional semantics and unsupervised part-of-speech induction algorithms, providing evidence that semantics and syntax exist on a continuum and emerge from a unitary cognitive system.

**Keywords:** semantic memory; mental lexicon; latent semantic analysis; statistical learning; holographic models; memory; cognitive models; semantic space.

### Orders of Association

Saussure (1916) defines two types of relationships between words: *paradigmatic* and *syntagmatic*. A syntagmatic relationship is the syntactic relationship a word has with other words that surround it. A paradigmatic relationship is when a pair of words can be substituted for each other. Building on Saussure, we define the term *order of association* as a measure of the degree of separation of two words in an agent's language experience.

**First order association** is when two words appear together. In the sentence "eagles soar over trees", the words *eagles* and *trees* have first order association. Words with strong first order association (i.e., frequently appear in the same sentence) are often related in topic.

**Second order association** is when two words appear with the same words. In the sentences "airplanes soar through skies" and "airplanes fly through skies", *soar* and *fly* have second order association. Words with strong second order association are often synonyms, or have a paradigmatic relationship in Saussure's terms.

**Third order association** is when two words appear with words that appear with the same words. Given the sentences in Table 1, the words *eagles* and *birds* have neither first nor second order association, but do have third order.

One can keep abstracting to higher-level orders of association indefinitely. At sufficiently higher orders of association, all words are related to all other words. Sensitivity to increasingly higher-order associations allows

one to identify increasingly abstract relationships between items, such as syntactic categories. We hypothesize that to properly capture the syntagmatic relationships between words in the English language, it is necessary for a cognitive model to be sensitive to at least third-order associations.

Table 1: Artificial data set for Simulation 1.

---

Sentences
eagles soar over trees
birds fly above forest
airplanes soar through skies
airplanes fly through skies
airplanes glide through skies
dishes are over plates
dishes are above plates
dishes are atop plates
squirrels live in trees
squirrels live in forest
squirrels live in woods

---

### Distributional Models of Semantics

Computational models of distributional semantics in the literature, such as LSA (Landauer & Dumais, 1997), HAL (Burgess & Lund, 1997), MINERVA 2 (Kwantes, 2005), the Topics Model (Griffiths, Steyvers, & Tenenbaum, 2007), and BEAGLE (Jones & Mewhort, 2007), are sensitive to only first and second-order associations. Jones and Mewhort observe clusters of vectors in semantic space that seem to correspond, roughly, to part of speech of information. Such clusters are suggestive of higher order associations, though BEAGLE does not exploit these higher order associations.

We have developed a model capable of detecting associations of arbitrarily high order. Using BEAGLE (Jones & Mewhort, 2007; but see also Kelly, Kwok, & West, 2015; Rutledge-Taylor et al., 2014, for variants on the model), a holographic model (Plate, 1995) of semantic memory, as a basis, we present a hierarchical model that layers multiple BEAGLE models. The *memory vector* outputs of one layer serve as the *environment vector* inputs to the next layer. The model roughly resembles a deep neural network in structure, but unlike a neural network, the model is not trained and the data is not subject to dimensional reduction at higher layers.

### Simulation 1

Higher layers of the model are sensitive to higher orders of association (Figures 1 and 2), as demonstrated by an artificial data set (Table 1). The memory vectors for words

with second order association, such as *soar* and *fly*, are close on Layer 1 (cosine = 0.44) and draw closer in higher layers (Layer 4, cosine = 0.71). Whereas *eagle* and *bird*, which have only third order association, are distant on Layer 1 (see Figure 1, cosine = -0.08) but are close on Layer 2 (cosine = 0.16) and draw closer in higher layers (see Figure 2, Layer 4, cosine = 0.47). Figures show cosine distances between 256 dimensional vectors, the distances compressed to 3 dimensions by multi-dimensional scaling.

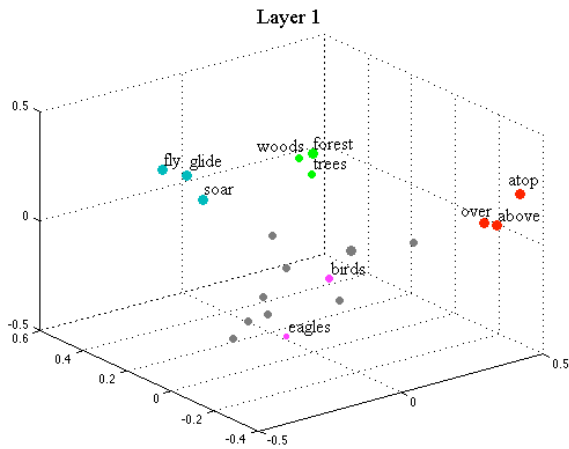


Figure 1: Cosine distances between vectors for Layer 1.

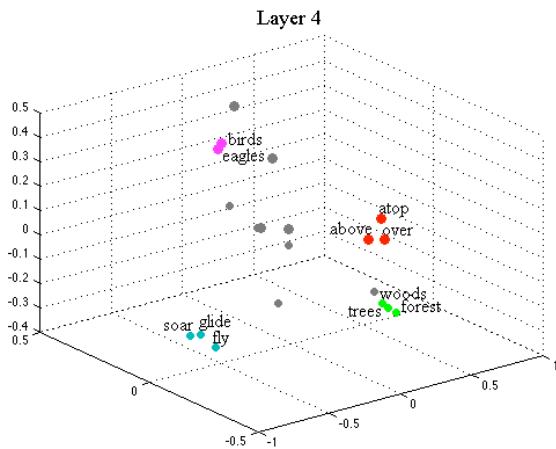


Figure 2: Cosine distances between vectors for Layer 4.

### Simulation 2

We ran the model on 15 out of copyright books from the *The Bobbsey Twins* series of children’s novels, available through Project Gutenberg. The corpus consists of 441 476 words and 9062 unique words. The model was run using 256 dimensional vectors and four layers. The model read the corpus one sentence at a time. Within each sentence, the model used a moving window of 21 words, 10 words to the left and right of a target word. Within that window, all *n-grams* are encoded as convolutions of environment vectors and summed into the target word’s memory vector.

We find that higher layers of the model exploit higher-order associations to strengthen semantically and syntactically correct relationships only weakly present in Layer 1 (see Figure 3) and to suppress erroneously strong relationships present in Layer 1 (see Figure 4).

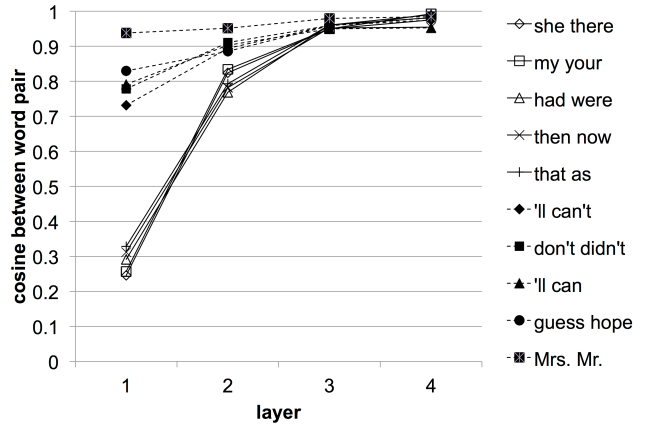


Figure 3: Cosine similarities between word pairs.

Word pairs are selected from the top 100 most similar word pairs on Layer 3 (Figure 3) or Layer 1 (Figure 4). Open marker and solid line indicates the 5 word pairs that increased in similarity the most from Layer 1 to Layer 3. Filled marker and dotted line indicates the 5 word pairs that increased in similarity the least from Layer 1 to Layer 3.

In Figure 3, we see a dramatic increase in similarity between word pairs such as the determiners *my* and *your*, from a cosine of 0.26 at Layer 1 to 0.99 at Layer 4. Conversely, word pairs such as the verbs *'ll* (contraction of *will*) and *can* or the titles *Mr.* and *Mrs.* are already highly similar at Layer 1 and so increase little across layers.

In Figure 4, we see that the similarity between dialogue tags *exclaimed* and *said* or the proper names of the twins *Bert* and *Nan* are strengthened by the higher order associations, whereas the erroneous relationship between the adjective *few* and *burgulor* (sic) or *thank* and the contraction *where'd* is suppressed.

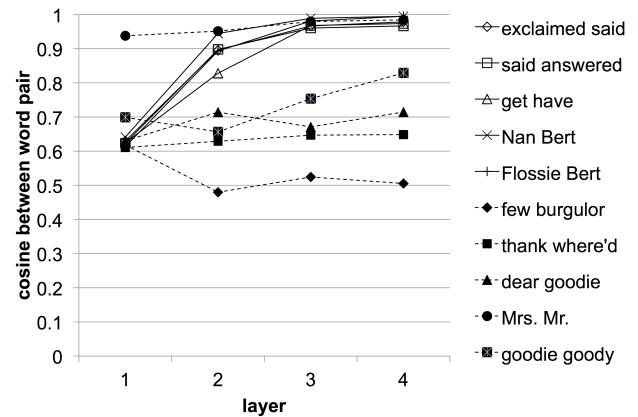


Figure 4: Cosine similarities between word pairs.

## Acknowledgments

This research is supported by an Ontario Graduate Scholarship awarded to the first author and a grant from the Natural Sciences and Engineering Research Council of Canada to the second author.

## References

- Burgess, C., & Lund, K. (1997). Modelling parsing constraints with high-dimensional context space. *Language and Cognitive Processes, 12*, 177-210.
- Jones, M. N., & Mewhort, D. J. K. (2007). Representing word meaning and order information in a composite holographic lexicon. *Psychological Review, 114*, 1-37. doi: 10.1037/0033-295X.114.1.1
- Griffiths, T. L., Steyvers, M., & Tenenbaum, J. B. (2007). Topics in semantic representation. *Psychological Review, 114*, 211-244. doi: 10.1037/0033-295X.114.2.211
- Kelly, M. A., Kwok, K., West, R. L. (2015). Holographic declarative memory and the fan effect: A test case for a new memory model for ACT-R. In N. A. Taatgen, M. K. van Vugt, J. P. Borst, & K. Mehlhorn (Eds.), *Proceedings of the 13th International Conference on Cognitive Modeling (pp. 148-153)*. Groningen, the Netherlands: University of Groningen.
- Kwantes, P. J. (2005). Using context to build semantics. *Psychonomic Bulletin & Review, 12*, 703-710.
- Landauer, T. K., & Dumais, S. T. (1997). A solution to Plato's problem: The latent semantic analysis theory of acquisition, induction, and representation of knowledge. *Psychological Review, 104*, 211-240.
- Plate, T. A. (1995). Holographic reduced representations. *IEEE Transactions on Neural Networks, 6*, 623-641. doi: 10.1109/72.377968
- Rutledge-Taylor, M. F., Kelly M. A., West, R. L., & Pyke, A. A. (2014). Dynamically structured holographic memory. *Biologically Inspired Cognitive Architectures, 9*, 9-32. doi: 10.1016/j.bica.2014.06.001
- Saussure, F. (1916). *Cours de linguistique générale*. C. Bally, A. Sechehaye, & A. Riedlinger, (Eds.). Lausanne, France: Payot.
- Feigenbaum, E. A. (1963). The simulation of verbal learning behavior. In E. A. Feigenbaum & J. Feldman (Eds.), *Computers and thought*. New York: McGraw-Hill.