Quantum Entanglement, Weak Measurements and the Conjunction and Disjunction Fallacies

Torr Polakow (torrpolakow@gmail.com)

Goren Gordon (goren@gorengordon.com)
Curiosity Lab, Department of Industrial Engineering
Sagol School Of Neuroscience, Tel-Aviv University, Israel, 6997801

Abstract

The conjunction and disjunction fallacies are expressions of irrational judgments. We propose a quantum cognition model that represents each concept as a separate qubit and the measurement process as a weak measurement. Using an on-line questionnaire, we analyzed the relation between irrational judgment and the corresponding quantum state entanglement. The model enables us to follow an individuals' quantum cognitive representation throughout the questionnaire and shows that, on average, participants get more entangled as they progress in the questionnaire. Our model accounts for multiple concepts simultaneously for rational and irrational decisions and suggests that quantum entanglement of mental concepts is correlated with irrational judgments.

Keywords: Quantum cognition; Irrationality; Quantum entanglement; Probability judgment fallacies.

Introduction

People tend to make irrational decisions (Tversky and Kahneman, 1983). Irrational behavior is any behavior that reflects a violation of basic laws that stem from classical probability theory (Kolmogorov, 2013). In this paper, we focus on the conjunction and disjunction fallacies, which violate the law of total probability: The conjunction fallacy occurs when a person judges the probability of the conjunction of two events to be more likely than either of the constituent events. The disjunction fallacy occurs when a person judges the probability of the disjunction of two events to be less likely than either of the constituent events. Quantum cognition is a developing field (Busemeyer and Bruza, 2012) that takes methods and concepts from quantum probability theory and uses them to explain and model decision-making findings. The hypothesis behind quantum cognition models is that irrational behavior obeys the laws of quantum theory rather than classical probability theory.

Studies of irrational behavior using classical methods have shown that people violate the unicity principle. This assumption is broken as soon as we allow incompatible questions into the theory, which causes measurements to be non-commutative. Incompatible questions cannot be evaluated on the same basis, so they require setting up separate sample spaces. This leads to conjunction and disjunction fallacies (Tversky and Kahneman, 1983; Tversky and Shafir, 1992). Quantum probability does not assume the principle of unicity, thus allowing one to use a partial Boolean algebra; each set of questions can be answered using one sample space in a Boolean fashion. All Boolean sub-algebras are pasted together in a coherent but non-Boolean fashion.

Previous work has shown that quantum probability (QP) can be used to model cognitive fallacies, specifically, conjunction and disjunction (Trueblood and Busemeyer, 2011; Pothos and Busemeyer, 2009; Franco, 2016). Two concepts analyzed in the fallacies lie in the same Hilbert space and represent two different reference frames. This is a framework that can account for the irrationalities but not for the rational behavior.

Quantum entanglement is a unique quantum phenomenon wherein two systems cannot be described as two *separable* systems. The only way to describe their joint quantum state is by describing it as a whole (Stolze and Suter, 2004; Salimi et al., 2012). Quantum entanglement has been used in the quantum interaction community to describe joint concepts (Grdenfors, 2004; Nelson and McEvoy, 2007; Bruza et al., 2009), albeit only when considering the population and not individual participants.

The theoretical framework of quantum weak measurements describes a quantum system by a generalized quantum state that propagates from the future as well as from the past (Aharonov and Vaidman, 1991). By ”pre-selecting” an initial quantum state and “post-selecting” a final quantum state, one can describe the full dynamics of a quantum system, sometimes enabling a description of peculiar phenomena, such as the Aharonov–Bohm effect (Aharonov and Bohm, 1959). This type of measurement is called “weak measurement” and will be exploited in our proposed quantum cognitive model.

We propose a quantum model based on entanglement and weak measurements that can account for rational and irrational behaviors as well as the dynamics of the mental state of the participants. Previous studies have not addressed the dynamics of irrationality, nor have they examined entanglement–irrationality relations. In this study, an on-line questionnaire (see Methods) containing several instantiations of conjunction and disjunction fallacy scenarios was administered. Using our model, we first show that it describes *all* participant results, i.e., both rational and irrational, for all the questions. We then show that our model enables an analysis of the multi-qubit quantum mental operations of each participant regarding each question, namely, a quantitative measure of bipartite quantum entanglement.

Our analysis of the survey data reveals that irrational judgment is represented by an entangled quantum state, whereas a separable quantum state represents a rational judgment in both the conjunction and disjunction fallacies. Finally, our model enables the analysis of *dynamics* throughout the questionnaire for each participant. We show that as more information is revealed about the concepts, the more entangled these concepts become. This formulation enables a more generic, scalable and intuitive representation of cognitive concepts.

Methods

Conjunction and disjunction fallacies. In our formalism, for two concepts A and B: (i) The conjunction fallacy occurs when $p\,(A\cap B) > \min(p(A), p(B))$, i.e., when the probability of the conjunction is greater than either of the constituent probabilities. (ii) The disjunction fallacy occurs when $p\,(A\cup B) < \max\,(p(A), p(B))$, i.e., when the probability of the disjunction is smaller than either of the constituent probabilities. We defined the irrationality measure as follows:

$$irr = p\,(A\cap B) - \min\,(p(A), p(B)) \qquad \text{conj. (1)}$$

$$irr = \max\,(p(A), p(B)) - p\,(A\cup B) \qquad \text{disj. (2)}$$

In the analysis, we defined an answer as irrational only if $irr > 0.1$, i.e., this is a stricter condition for irrationality.

Questionnaire. To study the conjunction and disjunction fallacies, we used the following personality sketches of two fictitious individuals, Emma and Liz, followed by a set of occupations and avocations associated with each or both of them. In each question, the participants were asked to give a probability for each option by using a horizontal slider/bar. All the options were initialized to the neutral probability value of 0.5.

We briefly outline the questionnaire as follows:

Q1: *Emma is outgoing and lives in an apartment within the center of the city with her two cats. She takes yoga classes at the gym three times a week, enjoys reading science-fiction books and volunteers in an animal shelter at least once a week. For each statement, please move the horizontal slider to represent how much do you think the statement represents Emma.*

Emma is a manger. $(= p(A))$
Emma is a pianist and a runner.
Emma is a writer.
Emma likes to paint.
Emma is vegan.
Emma likes to exercise. $(= p(B))$
Emma is a manger and likes to exercise. $(= p(A\cap B))$
Emma is a blogger.

Q2: *Liz lives in Oakland in a Victorian house. She is an analytical thinker and works in a start-up. In addition, she tries to go to a few classes at the gym every week. She is very ambitious in her job. She enjoys cooking very much and she is very good at it. She also likes camping. For each statement, please move the horizontal slider to represent your opinion about Liz.*

Is Liz a programmer? $(= p(C))$
Does Liz like to paint? $(= p(D))$
Is Liz a programmer and likes to paint? $(= p(C\cap D))$

Q3: *Emma and Liz want to do an extracurricular activity together.For each statement, please move the horizontal slider to represent how likely it is they will choose this activity and why.*

Take spinning class, because Emma likes to exercise. $(= p(B))$
Try out gourmet restaurants in the city.
Take realistic painting classes, because Liz like to paint. $(= p(D))$
Take singing classes near Emma's apartment.
Take photography class near Liz house.
Take spinning class or realistic painting classes. $(= p(B \cup D))$

Q4: *Recently Emma got to the conclusion that she doesn't have enough time in a day. For each statement, please move the horizontal slider to represent how likely it is that this is the reason Emma needs more time.*

For her work, as manager. $(= p(A))$
Volunteer.
Exercise. $(= p(B))$
Join Book club, because she likes to read.
To work as manager or to exercise. $(= p(A \cup B))$
Meet with friends.

Participants were 100 Amazon Mechanical Turk users. We asked for Amazon Mechanical Turk Masters that were native English speakers from North America that had completed at least 100 tasks with an approval rate $> 95\%$. Each participant was presented with the questionnaire on Qualtrics. After completing the questionnaire, the participants had to pass three screening checks:

a) "Trap" question - we inserted a question that contained internal text telling the participant how to answer. Participants that answered incorrectly were excluded (15 participants). **b)** Response time - participants who answered too quickly/slowly, i.e., more than a 3σ deviation from the mean response time in either direction were excluded (8 participants). **c)** "Focus" - Participants that answered too many questions (3σ deviation from the mean "focus") with the probabilities $0, 0.5, 1$, i.e., they did not pay attention to the answers, were excluded (2 participants).

After screening, 78 participants were left (there were participants who failed more than one screening test).

Results

Participant Irrationality

We first present the data from the on-line survey we performed, Fig. 1. The survey included four questions; the first and second measured the conjunction fallacy, while the third and fourth measured the disjunction fallacy. As can be seen, the percentage of irrational judgments replicates previous reporting (Charness, 2009). Furthermore, only 8 out of the 78 participants were rational in all questions.

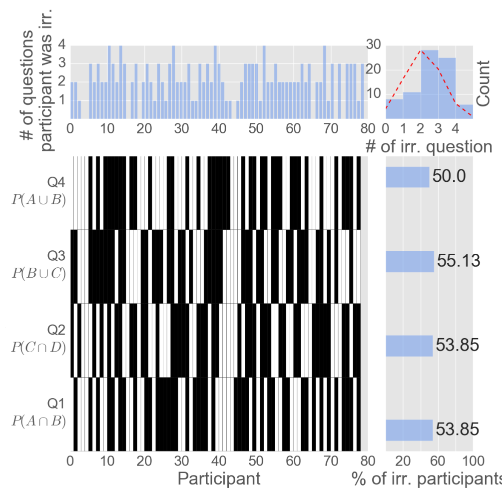

Figure 1: Top-left: Number of irrational answers per participant; Bottom-left: (ir)rational answers (white=rational, black=irrational); Top-right: histogram of the number of irrational answers per participant; Bottom-right: distribution of the irrationality value per question.

These data suggest that a questionnaire involving multiple questions and different types of fallacies can reveal the ubiquity of irrational judgments.

Moreover, as detailed in the next sections, analyzing each participant individually throughout the questionnaire enables a glimpse into the *dynamics* of irrational decision making. For example, comparing the first and last questions, which asked about the same concepts, reveals that only 45 participants (58%) maintained their "rationality", i.e., answered both questions (ir)rationally. While this can be interpreted as inconsistency, the participants were given more information about the concepts between the two questions and thus may represent a dynamic mental process (see below).

Weak Measurements of Concept-Qubits

In our model, we propose that each concept is represented by a single qubit: Concept A is represented by $|\psi\rangle_A$, while a different concept, B is represented by another qubit, $|\psi\rangle_B$. Thus, the complete two-concept quantum state is represented by

$$
\begin{aligned}
|\psi\rangle_{AB} &= a_{00}|0\rangle_A|0\rangle_B + a_{10}|1\rangle_A|0\rangle_B + \\
&\quad a_{01}|0\rangle_A|1\rangle_B + a_{11}|1\rangle_A|1\rangle_B \quad (3)\\
|\sum_{ij} a_{ij}|^2 &= 1 \quad (4)\\
P(A) &= Tr_B({}_A\langle 1|\psi\rangle_{AB}\langle\psi|\psi_0\rangle_A) \quad (5)\\
&= (1/\sqrt{2})(a_{10}(a_{00}+a_{10}) + a_{11}(a_{01}+a_{11}))\\
P(B) &= Tr_A({}_B\langle 1|\psi\rangle_{AB}\langle\psi|\psi_0\rangle_B) \quad (6)\\
&= (1/\sqrt{2})(a_{01}(a_{00}+a_{01}) + a_{11}(a_{10}+a_{11}))\\
P(A\cap B) &= {}_A\langle 1|_B\langle 1|\psi\rangle_{AB}\langle\psi|\psi_0\rangle_B|\psi_0\rangle_A \quad (7)\\
&= (1/2)a_{11}(a_{00}+a_{10}+a_{01}+a_{11})\\
P(A\cup B) &= ({}_A\langle 1|_B\langle 0| + {}_A\langle 0|_B\langle 1| + {}_A\langle 1|_B\langle 1|)\times\\
&\quad \psi\rangle_{AB}\langle\psi|\psi_0\rangle_B|\psi_0\rangle_A \quad (8)\\
&= (1/(3\sqrt{2}))(a_{10}+a_{01}+a_{11})\times\\
&\quad (a_{00}+a_{10}+a_{01}+a_{11})
\end{aligned}
$$

This representation has three free parameters due to normalization (eq. (4)) as compared to the two parameters of previous models (Trueblood and Busemeyer, 2011; Pothos and Busemeyer, 2009; Franco, 2016). For each participant and each conjunction (disjunction) question, we obtain three reported probabilities, namely, $p(A), p(B)$ and $p(A\cap B)$ (or $(p(A\cup B))$.

We introduce *weak measurements* as the measurement process in our model (Aharonov and Vaidman, 1991). For new questions regarding concepts about which there is no information, the pre-selected state is given by the fully superposed state $|\psi_0\rangle = 1/\sqrt{2}(|0\rangle + |1\rangle)$. The post-selected state is the answer in the questionnaire, in our case, always $|1\rangle$ of the relevant concept-qubit. The mental quantum operation each participant performs in each question transforms the initial state to the final one. This is represented by $|\psi\rangle_{AB}\langle\psi|$. In other words, the participants' mental process of how they incorporate new information is represented by a projection operator.

This model enables the calculation of the full quantum mental state representation given the reported question probabilities. Under the formalism from eq. (3) we denote the constraints eqs. (4) and (8).

For the conjunction questions, we used eqs. (4)–(7)), and for the disjunction questions, we used eqs. (4)–(6) and (8)). We numerically solved this set of four non-linear equations with four variables, which resulted in a full quantum state for each participant and each question.

Entanglement and Irrationality

While the calculation of the full quantum state from the probabilities does not generate any prediction, it does enable us to calculate entanglement. We calculated the two-qubit pure-state entanglement using the concurrence measurement (Stolze and Suter, 2004):

$$C(|\psi\rangle) = 2\cdot|a_{00}\cdot a_{11} - a_{01}\cdot a_{10}| \quad (9)$$

where $C\in[0,1]$, so that if $C=0$, the state is factorized, whereas if $C>0$, the state is entangled. In the data analysis, we defined a stricter threshold for entanglement, namely, a state representing a participant's answer is considered entangled only if $C>0.2$.

This quantum entanglement calculation enables us to analyze its relation to the amount of irrationality of the reported probabilities, eq. (1). Hence, we can compute both entanglement and irrationality for each participant and each question , as shown in fig. 1.

As can be seen in Fig. 2, with our strict definitions of irrationality and the entangled state, only two out of the 78 participants were both irrational and non-entangled in this question. Fig. 3 shows that this "quadrant" was sparse in all questions, i.e., out of all the participants/questions (312 in total), only 5.8% (18 answers) were irrational and separable. More quantitatively, we can compute the following conditional probabilities:

$$P(\text{rational}|\text{low entanglement}) = 75\% \quad (10)$$

$$P(\text{high entanglement}|\text{irrational}) = 81\% \quad (11)$$

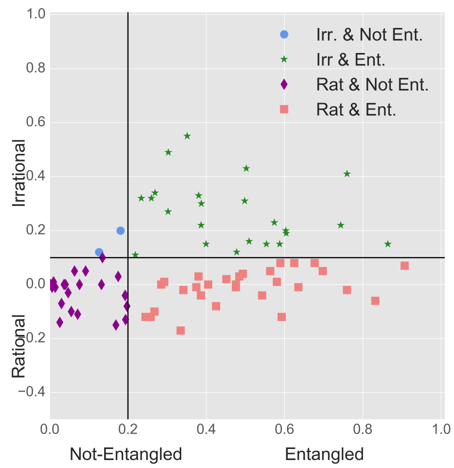

Figure 2: Irrationality as a function of Entanglement (concurrence calculation).

Finally, the questionnaire enables us to follow the dynamics of irrational judgments and the ensuing entanglement. As can be seen in Fig. 3, entanglement is monotonically non-decreasing as the questionnaire progresses. This is expected as more information regarding the concepts is revealed, i.e., as the story of Emma and Liz unfolds. The intricate connections between the storylines generate a quantum entanglement of the representative quantum states.

Discussion and Future Work

We have presented a quantum model with respect to the conjunction and disjunction fallacies that represents each concept as a separate qubit and treats the questions as quantum weak measurements.

While previous quantum cognitive models have treated concepts as qubits, (Busemeyer and Bruza, 2012), they have done so on overall data, i.e., by aggregating answers from many participants, thus representing the "concepts" as a whole. In contrast, our model attempts to represent *individual* mental states of participants by fitting their answers to a specific quantum projection operator within the weak-measurement framework. This framework assumes that participants start with an ignorant representation of the concept, represented as a full superposition of all possible representations as the "preselected" quantum state. The framing of the question "post-selects" the end quantum state, which enables us to fit the quantum operator, represented as a projection of a full quantum state of both concepts, from the data.

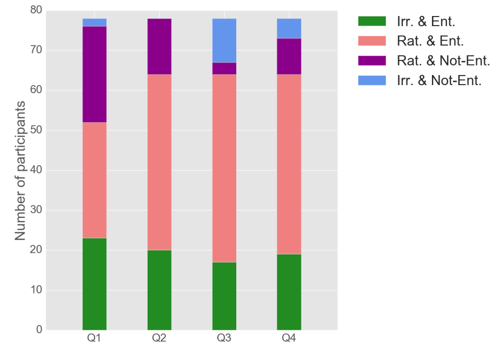

Figure 3: Distribution of rational/irrational and entangled/non-entangled participants for all the questions.

This representation gives new insights into the connection between the quantum mental representation of concepts and irrational judgments. More specifically, the data suggest that irrational judgments mostly occur for entangled quantum states, whereas separable states occur mostly when rational behavior is observed.

Cognitively, one can speculate that rational judgment regarding two concepts implies that they are separable and do not relate to or influence each other. This separability thus conveys no "cognitive interference" that can cause irrational judgments. On the other hand, highly entangled concepts, i.e., concepts that relate to and influence each other in a tight manner, will result in more irrational judgments.

Our participant–question-specific model enables us to analyze the *dynamics* of entanglement throughout the questionnaire. As hypothesized by the connection between concepts and entanglement, the more information is revealed throughout the questionnaire regarding the concepts, the more entangled they become. We have shown that entanglement indeed rises on average, and more specifically, that for the same question, for answers at the beginning and the end of the questionnaire, entanglement increased.

The proposed model holds promise in the form of scalability. The previous concept-as-basis models did not scale well when introducing more than two concepts, since introducing even a single new concept immediately imposes *two* relations between the previous two concepts. This occurs since all concepts lie in the same Hilbert space. Our concept-as-qubit model enables the introduction of more concepts, as they expand the Hilbert space and enable arbitrary relations between the concepts. While the increase in free parameters is exponential in the number of concepts in our model, measures can be computed from the inferred quantum state; the most promising are multipartite entanglement measures. Future work will explore this direction with a more detailed questionnaire that involves more than two qubits and their interaction.

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