Decoy Effect and Violation of Betweenness in Risky Decision Making: A Resource-Rational Mechanistic Account

Ardavan S. Nobandegani[†,1,3], **Kevin da Silva Castanheira**[†,3], **A. Ross Otto**[3], **& Thomas R. Shultz**[2,3]
{ardavan.salehinobandegani, kevin.dasilvacastanheira}@mail.mcgill.ca
{ross.otto, thomas.shultz}@mcgill.ca
[1]Department of Electrical & Computer Engineering, McGill University
[2]School of Computer Science, McGill University
[3]Department of Psychology, McGill University
[†]**Co-primary authors**

Abstract

A wealth of experimental evidence shows that, contrary to normative models of choice, people's preferences are markedly swayed by the context in which options are presented. In this work, we present the first resource-rational, mechanistic account of the decoy effect—a major contextual effect in risky decision making. Our model additionally explains a related, well-known behavioral departure from expected utility theory: violation of betweenness. We demonstrate that, contrary to widely held views, these effects can be accounted for by a variant of normative expected-utility-maximization, which acknowledges cognitive limitations. Our work is consistent with two empirically well-supported hypotheses: (i) In probabilistic reasoning and judgment, a cognitive system accumulates information through sampling, and (ii) People engage in pairwise comparisons when choosing between multiple alternatives.

Keywords: Risky decision-making; decoy effect; violation of betweenness; rational process models; expected utility theory

1 Introduction

Expected utility theory (EU), the most prominent model of rational choice (Bernoulli, 1738/2011), maintains that people's preferences should not change depending on the context in which options are presented. More specifically, according to the von Neumann-Morgenstern axiomatization of EU (von Neumann & Morgenstern, 1947/2007), a rational decision-maker obeys the independence axiom: preferences between alternatives A and B depend only on preferences between A and B. Formally, the independence axiom prescribes the following: If A is preferred to B out of the choice set $\{A,B\}$, introducing a third option X, expanding the choice set to $\{A,B,X\}$, does not make B preferable to A.

Contrary to the independence axiom, however, a wealth of experimental evidence shows that people's preferences are markedly swayed by the context in which options are presented (e.g., Huber, Payne, & Puto, 1982; Wedell, 1991; Roe, Busemeyer, & Townsend, 2001; Soltani, De Martino, & Camerer, 2012; Tsetsos, Chater, & Usher, 2012; Noguchi & Stewart, 2014; Mohr, Heekeren, Rieskamp, 2017).

Although contextual effects are predominantly studied in the realm of multi-attribute decision making without risk (e.g., Roe et al., 2001; Noguchi & Stewart, 2014), several studies have experimentally investigated contextual effects in risky decision-making (Huber et al., 1982; Wedell, 1991; Soltani et al., 2012; Tsetsos et al., 2012; Mohr et al., 2017).

A prominent contextual effect in risky choice is the decoy effect (e.g., Mohr et al., 2017) according to which the inclusion of a third asymmetrically-dominated gamble (decoy) into the choice set leads to increased preference for the dominating gamble (target), thus clearly violating the independence axiom of EU.

In this work, we present the first resource-rational, mechanistic account of the decoy effect in risky decision-making. Concretely, we show that, contrary to widely held views, this effect can be accounted for by a variant of the normative maximizing of expected utility, *sample-based expected utility* (SbEU), which acknowledges cognitive limitations that a decision-maker is faced with (Nobandegani, da Silva Castanheira, Otto, & Shultz, 2018).

SbEU is a metacognitively-rational, process model that takes into account that people adapt their strategies depending on the amount of time available for decision-making (e.g., Maule & Svenson, 1993; Svenson, 1993). Consistent with a large body of evidence, SbEU posits that, in probabilistic reasoning and judgment, a cognitive system accumulates information through *sampling* (e.g., Vul et al., 2014; Battaglia et al., 2013; Lake et al., 2017; Gershman, Horvitz, & Tenenbaum, 2015; Hertwig & Pleskac, 2010; Griffiths et al., 2012; Gershman, Vul, & Tenenbaum, 2012; Bonawitz et al., 2014).

Additionally, our mechanistic explanation of the decoy effect relies on a key assumption: people engage in pairwise comparisons when choosing between multiple alternatives. Recent experimental work has provided mounting evidence for this assumption (e.g., Russo & Rosen, 1975; Noguchi & Stewart, 2014). Specifically, recent eye-tracking work by Noguchi and Stewart (2014) shows that, when choosing between multiple alternatives, a series of comparisons is made in each choice, with a pair of alternatives compared on a single attribute dimension in each comparison.

We furthermore show that our resource-rational, process-level account of the decoy effect can also explain another related, well-known behavioral departure from EU: violation of betweenness (e.g., Camerer & Ho, 1994; Prelec, 1990). Concretely, betweenness is a weakened form of the independence axiom, prescribing that a probability mixture of two risky gambles should lie between them in preference (Camerer & Ho, 1994). Despite being widely assumed in game theory, auction theory, macroeconomics, and dynamic choice, violations of betweenness are experimentally well-documented (e.g., Camerer & Ho, 1994; Prelec, 1990).

After presenting a brief overview of SbEU, we proceed to model the decoy effect and violation of betweenness in risky decision-making.

2 Sample-based Expected Utility Model

Extending the cognitively-rational decision-making model of Lieder, Griffiths, and Hsu (2018) to the realm of metacognition (Roberts & Erdos, 1993; Cary & Reder, 2002), SbEU is a metacognitively-rational process model of risky choice, positing that an agent rationally adapts their strategies depending on the amount of time available for decision-making (Nobandegani et al., 2018). Concretely, SbEU assumes that an agent estimates expected utility

$$\mathbb{E}[u(o)] = \int p(o)u(o)do, \tag{1}$$

using self-normalized importance sampling (Hammersley & Handscomb, 1964; Geweke, 1989), with its importance distribution q^* aiming to minimize mean-squared error (MSE):

$$\hat{E} = \frac{1}{\sum_{j=1}^{s} w_j} \sum_{i=1}^{s} w_i u(o_i), \quad \forall i:\ o_i \sim q^*,\ w_i = \frac{p(o_i)}{q^*(o_i)}, \tag{2}$$

$$q^*(o) \propto p(o)|u(o)|\sqrt{\frac{1+|u(o)|\sqrt{s}}{|u(o)|\sqrt{s}}}. \tag{3}$$

MSE is a standard normative measure of the quality of an estimator, and is widely adopted in machine learning and mathematical statistics (Poor, 2013). In Eqs. (1-3), o denotes an outcome of a risky gamble, $p(o)$ the objective probability of outcome o, $u(o)$ the subjective utility of outcome o, $\hat{E}$ the importance-sampling estimate of expected utility given in Eq. (1), q^* the importance-sampling distribution, o_i an outcome randomly sampled from q^*, and s the number of samples drawn from q^*.

While cognitively-rational agents are ignorant about adapting their importance distribution q based on time availability, a *meta*cognitively-rational agent would plausibly use such considerations in their choice of q. That is, the metacognitively-rational agent chooses a q which is normatively-justified based on time availability considerations, allowing strategy selection to be guided by time availability. In agreement with this view, a large body of psychological work on decision-making suggests that people adapt their strategies in accord with time availability (e.g., Maule & Svenson, 1993; Svenson, 1993). As evidenced by Eq. 3 explicitly depending on s, SbEU assumes that decision-makers rationally adapt their strategies depending on time availability.

SbEU posits that, when choosing between a pair of risky gambles $\{A,B\}$, people make their choice depending on whether the expected value of the utility difference $\Delta u(o)$ is negative or positive (w.p. stands for with probability):

$$A = \begin{cases} o_A & \text{w.p. } p_A \\ 0 & \text{w.p. } 1-p_A \end{cases} \tag{4}$$

$$B = \begin{cases} o_B & \text{w.p. } p_B \\ 0 & \text{w.p. } 1-p_B \end{cases} \tag{5}$$

$$\Delta u(o) = \begin{cases} u(o_A) - u(o_B) & \text{w.p. } p_A p_B \\ u(o_A) - u(0) & \text{w.p. } p_A(1-p_B) \\ u(0) - u(o_B) & \text{w.p. } (1-p_A)p_B \\ 0 & \text{w.p. } (1-p_A)(1-p_B) \end{cases} \tag{6}$$

Recent work by Nobandegani et al. (2018) showed that SbEU can account for availability bias, people's tendency to overestimate the probability of events that easily come to mind (Tversky & Kahneman, 1973), and can accurately simulate the well-known fourfold pattern of risk preferences in outcome probability (Tversky & Kahneman, 1992) and in outcome magnitude (Markovitz, 1952; Hershey & Schoemaker, 1980; Scholten & Read, 2014). Notably, SbEU is the first rational process model to score near-perfectly in optimality, economical use of limited cognitive resources, and robustness, simultaneously (Nobandegani et al., 2018; Nobandegani, da Silva Castanheira, O'Donnell, & Shultz, 2019).

Following Nobandegani et al. (2018), and consistent with prospect theory (Kahneman & Tversky, 1979) and cumulative prospect theory (Kahneman & Tversky, 1992), in this work we assume the following S-shaped, utility function:

$$u(x) = \begin{cases} x^{0.85} & \text{if } x \geq 0, \\ -|x|^{0.95} & \text{if } x < 0. \end{cases} \tag{7}$$

3 Decoy Effect in Risky Decision-Making

A prominent contextual effect in risky choice is the decoy effect (DE) according to which the inclusion of a third asymmetrically-dominated gamble (decoy D) into the choice set $\{T,C\}$ (comprising of target T and competitor C) leads to increased preference for the dominating gamble (target T), thus violating the independence axiom of EU (Arrow, 1963; Ray, 1973; Machina, 1987).

Formally, DE can be mathematically characterize by having $\mathbb{P}(T|\{T,C,D\}) > \mathbb{P}(T|\{T,C\})$ (Huber et al., 1982; Simonson, 1989; Speekenbrink & Shanks, 2013; Mohr et al., 2017), with $\mathbb{P}(T|\{T,C,D\})$ and $\mathbb{P}(T|\{T,C\})$ denoting the probability of choosing T when the choice set is $\{T,C\}$ and $\{T,C,D\}$, respectively.[1]

Consistent with mounting experimental evidence (e.g., Russo & Rosen, 1975; Noguchi & Stewart, 2014), we assume that the decision-maker engages in pairwise comparisons when choosing from the choice set $\{T,C,D\}$, with parameters p_{tc}, p_{cd}, p_{td} denoting the probability of starting with the pairs $(T,C),(C,D),(T,D)$, respectively. The winner of the first pairwise comparison will then compete against the remaining risky gamble. Ultimately, the winner of the final

[1]DE is also a violation of the regularity axiom—a weakened form of the independence axiom—according to which the addition of an option to the choice set can never increase the probability of choosing an option relative to the original set (Speekenbrink & Shanks, 2013). More formally, for options X,Y,Z, the regularity principle prescribes the following: $\mathbb{P}(X|\{X,Y\}) > \mathbb{P}(X|\{X,Y,Z\})$.

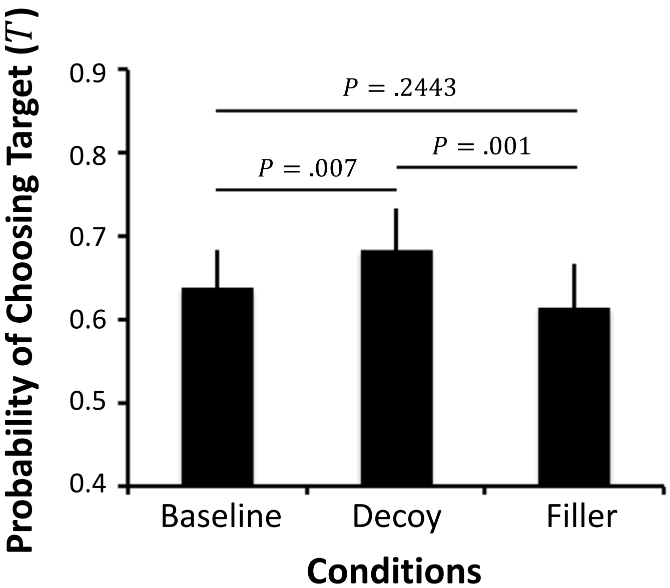

Figure 1: Mohr et al.'s (2017) experimental data. Once decoy (D) is added to the choice set (middle bar, Decoy Condition), people's preference for the target (T) significantly increases. People's preference for the target in the Decoy Condition is also significantly higher than it is in the Filler Condition. However, people's preference for the target is not significantly different between the Baseline Condition (wherein the choice set is $\{C,T\}$) and the Filler Condition (wherein the choice set is $\{C,T,F\}$). Adapted from Mohr et al.'s (2017, Fig. 2A).

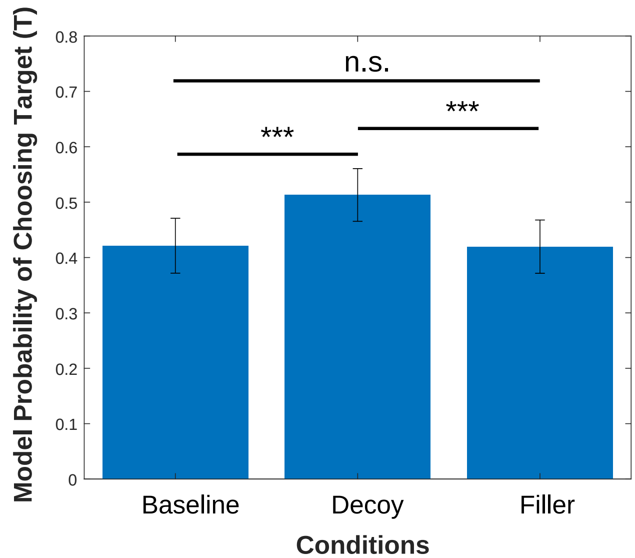

Figure 2: Model simulation of Mohr et al.'s (2017) experimental data reported in Fig. 1. Error bars indicate standard deviation (SD). $^{***}P < .001$, n.s. not significant.

pairwise comparison will determine the explicit choice that the agent makes from the original choice set $\{T,C,D\}$.

Recently, Mohr et al. (2017) investigated the neural underpinnings of DE in risky decision making using fMRI. Concretely, Mohr et al. (2017) showed that specific brain regions (e.g., the medial orbitofrontal cortex and the anterior insula) not only code the value or risk of a single choice option but also code the evidence in favor of the best option compared with other available choice options.

In their behavioral experiment, Mohr et al. (2017) showed that $\mathbb{P}(T|\{T,C,D\}) > \mathbb{P}(T|\{T,C\})$ when D is asymmetrically dominated (i.e., D is dominated by T but not C), while $\mathbb{P}(T|\{T,C\}) \approx \mathbb{P}(T|\{T,C,F\})$ when a gamble F (called filler) is dominated by both T and D, thus experimentally confirming DE in risky decision-making. Mohr et al.'s (2017) experimental data are shown in Fig. 1.

Next, we show that SbEU, together with the experimentally well-supported assumption of pairwise comparison, can provide a resource-rational mechanistic explanation of the behavioral finding by Mohr et al. (2017) discussed above. For our simulation of risky DE, we adopt a representative stimulus from Mohr et al. (2017, Fig. 1), involving four gambles (€ denotes the Euro sign):

$$C = \begin{cases} 80€ & \text{w.p. } 20\% \\ 0 & \text{w.p. } 80\% \end{cases}$$

$$T = \begin{cases} 20€ & \text{w.p. } 80\% \\ 0 & \text{w.p. } 20\% \end{cases}$$

$$D = \begin{cases} 20€ & \text{w.p. } 70\% \\ 0 & \text{w.p. } 30\% \end{cases}$$

$$F = \begin{cases} 20€ & \text{w.p. } 20\% \\ 0 & \text{w.p. } 80\% \end{cases}$$

where C,T,D,F denote the competitor, target, decoy, and filler gambles, respectively.

A la Mohr et al. (2017), we consider three experimental conditions, with the Baseline Condition, Decoy Condition and Filler Condition corresponding to participants choosing from the choice sets $\{T,C\},\{T,C,D\},\{T,C,F\}$, respectively.

We simulate $N = 1000$ participants, with every participant performing 100 trials of each of the experimental conditions. Model predictions for a few samples ($s = 4$) are shown in Fig. 2. This choice of s is supported by recent work providing mounting evidence that people often use only a few samples in probabilistic judgments and reasoning under uncertainty (e.g., Vul et al., 2014; Battaglia et al. 2013; Lake et al., 2017; Gershman, Horvitz, & Tenenbaum, 2015; Hertwig & Pleskac, 2010; Griffiths et al., 2012; Gershman, Vul, & Tenenbaum, 2012; Bonawitz et al., 2014; Nobandegani et al., 2018).

Fully consistent with Mohr et al.'s (2017) experimental results (see Fig.1), SbEU predicts that $\mathbb{P}(T|\{T,C,D\}) > \mathbb{P}(T|\{T,C\})$ ($t(999) = 42.2177$, $P < .001$, Cohen's $d = 1.3350$) and $\mathbb{P}(T|\{T,C,D\}) > \mathbb{P}(T|\{T,C,F\})$ ($t(999) =$

43.9820, $P < .001$, Cohen's $d = 1.3908$), while predicting that $\mathbb{P}(T|\{T,C\}) \approx \mathbb{P}(T|\{T,C,F\})$ ($t(999) = 0.7550$, $P = 0.4504$, Cohen's $d = 0.0239$). Model predictions are shown in Fig. 2.

In Fig. 2, we set $p_{tc} = 0.01, p_{cd} = 0.98, p_{td} = 0.01$. Recall that the parameters p_{tc}, p_{cd}, p_{td} denote the probability of starting with the pairs $(T,C), (C,D), (T,D)$, respectively. The relatively high value of p_{cd} receives strong theoretical supports from Theorem 1. Specifically, Theorem 1 provides a general, theoretical foundation for risky DE, under the experimentally well-supported hypothesis that people engage in pairwise comparisons when choosing between multiple alternatives (the pairwise-comparison hypothesis).

Theorem 1. *Let $S = \{T,C,D\}$ be the choice set, with T,C,D denoting the target, competitor, and decoy, respectively. Assuming that a decision-maker is to always start with a particular pair, then the following holds true: Starting only with the pair (C,D) can potentially produce risky DE. That is, starting with the pair (C,D) can potentially lead to having $\mathbb{P}(T|\{T,C,D\}) > \mathbb{P}(T|\{T,C\})$, while starting with the pair (T,C) or (T,D) grants $\mathbb{P}(T|\{T,C,D\}) \not> \mathbb{P}(T|\{T,C\})$.*

Proof of Theorem 1 is given in the Appendix. Theorem 1 has an important implication which can be articulated in simple terms as follows: Assuming that people perform pairwise comparisons when choosing between multiple alternatives, the only reliable way of producing risky DE is for people to significantly direct their attention to the pair (C,D) at the outset of their decision-making process. (Recall that the amount of attention directed at the pair (C,D) at the outset of decision-making is controlled by the parameter p_{cd}.) This provides a mathematically-rigorous, formal basis for our choice of $p_{tc} = 0.01, p_{cd} = .98, p_{td} = 0.01$.

4 Violation of Betweenness in Risky Choice

Betweenness is a relaxation of the independence axiom, prescribing that a probability mixture of two risky gambles should lie between them in preference (Camerer & Ho, 1994). Despite being widely assumed in game theory, auction theory, macroeconomics, and dynamic choice, experimental violations of betweenness are well-documented (e.g., Camerer & Ho, 1994; Prelec, 1990).

Formally, betweenness can be characterized as follows (Camerer & Ho, 1994): If risky gamble A is preferred to risky gamble B (i.e., $A \succ B$), then the following should hold: $\forall p \in (0,1): \; A \succ pA + (1-p)B \succ B$, where $pA + (1-p)B$ denotes a probabilistic mixture of A and B with probabilities p and $1-p$, respectively. In simple terms, betweenness requires that every probabilistic mixture of two gambles A and B lie between them in preference (hence the term "betweenness").

Next, we show that SbEU can additionally account for an experimentally-documented violation of betweenness (Prelec, 1990; Camerer & Ho, 1994).

An experiment by Prelec (1990), and replicated by Camerer and Ho (1994), revealed that people preferred X to Y, but preferred Z (a probabilistic mixture of X and Y) to X, hence violating the betweenness property. The gambles X, Y, Z are given by (Prelec, 1990):

$$X = \begin{cases} \$20{,}000 & \text{w.p. } 34\% \\ 0 & \text{w.p. } 66\% \end{cases} \quad Y = \begin{cases} \$30{,}000 & \text{w.p. } 17\% \\ 0 & \text{w.p. } 83\% \end{cases}$$

$$Z = \begin{cases} \$30{,}000 & \text{w.p. } 1\% \\ \$20{,}000 & \text{w.p. } 32\% \\ 0 & \text{w.p. } 67\% \end{cases}$$

where $Z = \frac{16}{17}X + \frac{1}{17}Y$, i.e., Z is a probabilistic mixture of the gambles X and Y with probabilities $\frac{16}{17}$ and $\frac{1}{17}$, respectively.

As we did in our simulation of risky DE, we simulate $N = 1000$ participants, with each participant performing 100 trials of each of the experimental conditions (Condition I: choosing between the gamble X and Y; Condition II: choosing between the gambles X and Z).

Model predictions for a few samples ($s = 4$) are shown in Fig. 3. Consistent with the Prelec's (1990) experimental data, SbEU predicts that, in Condition I, $\mathbb{P}(X|\{X,Y\}) > \mathbb{P}(Y|\{X,Y\})$ ($t(999) = 2.4160$, $P = .0159$, Cohen's $d = 0.0764$) while predicting that, in Condition II, $\mathbb{P}(Z|\{X,Z\}) > \mathbb{P}(X|\{X,Z\})$ ($t(999) = 44.8260$, $P < .001$, Cohen's $d = 1.4175$).

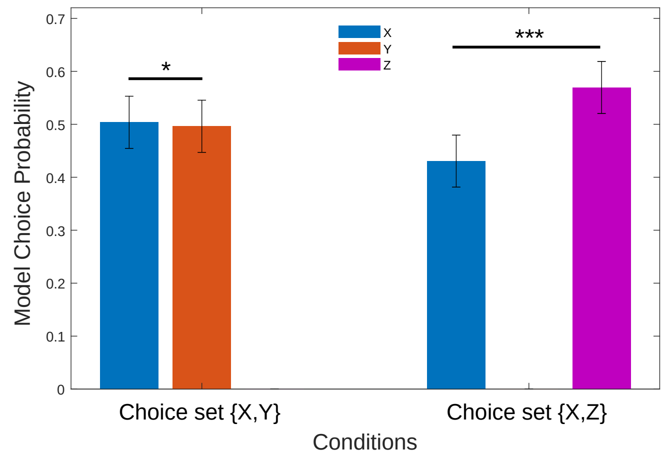

Figure 3: Model simulation of Prelec's (1990) experimental results. Consistent with the experimental data, the model predicts that, when choosing from the choice set $\{X,Y\}$, people prefer X over Y. However, when choosing from the choice set $\{X,Z\}$, the model predicts that people prefer Z over X, thus violating the betweenness property. Error bars indicate standard deviations.$^{*}P < .05$, $^{***}P < .001$.

5 General Discussion

Expected utility theory (EU), the most prominent model of rational choice, maintains that people's preferences should not change depending on the context in which options are presented. Contrary to a widely held view, in this work we show that a variant of normative expected-utility-maximization which acknowledges cognitive limitations, SbEU (Nobandegani et al., 2018), can provide a metacognitively-rational process-level account of a prominent contextual effect in risky decision-making: the decoy effect (e.g., Mohr et al., 2017). Additionally, our explanation provides a resource-rational mechanistic account of another behavioral departure from EU: violations of betweenness (e.g., Camerer & Ho, 1994; Prelec, 1990). Betweenness, a relaxation of the independence axiom, has played a prominent role in developing generalizations of EU and their applications to game theory and macroeconomics (Camerer & Ho, 1994).

Recent work has shown that SbEU can account for the St. Petersburg paradox, a centuries-old paradox in human decision-making (Nobandegani, da Silva Castanheira, Shultz, & Otto, 2019a), and can provide a resource-rational mechanistic account of (ostensibly irrational) cooperation in one-shot Prisoner's Dilemma games, thus successfully bridging between game-theoretic decision-making and risky decision-making (Nobandegani, da Silva Castanheira, Shultz, & Otto, 2019b). There is also experimental confirmation of a counterintuitive prediction of SbEU: Deliberation leads people to move from one well-known bias, framing effect, to another well-known bias, the fourfold pattern of risk preferences (da Silva Castanheira; Nobandegani, & Otto, 2019).

Notably, the present study is simultaneously guided by, and consistent with, two empirically well-supported assumptions: (1) In probabilistic reasoning and judgment, a cognitive system accumulates information through sampling (e.g., Vul et al., 2014; Battaglia et al. 2013; Lake et al., 2017; Gershman, Horvitz, & Tenenbaum, 2015; Hertwig & Pleskac, 2010; Griffiths et al., 2012; Gershman, Vul, & Tenenbaum, 2012; Bonawitz et al., 2014), and (2) People engage in pairwise comparisons when choosing between multiple alternatives (e.g., Russo & Rosen, 1975; Noguchi & Stewart, 2014).

Camerer and Ho (1994) provide evidence suggesting that people are more likely to violate the independence and the betweenness axioms when presented with single-stage gambles than with multi-stage gambles (wherein, with some probability, the agent is presented with one risky gamble, and, with some other probability, with a different risky gamble). The present study particularly focused on single-stage gambles. Future work should investigate if the resource-rational process-level explanation presented in this work could also account for the forgoing tendency experimentally documented by Camerer and Ho (1994).

There have been several recent studies (see Lieder & Griffiths, 2018, for a review) attempting to show that many well-known (purportedly irrational) behavioral effects and cognitive biases can be understood as optimal behavior subject to computational and cognitive limitations (e.g., Griffiths, Lieder, & Goodman, 2015; Nobandegani, 2018; Lieder, Griffiths, Huys, & Goodman, 2018). The present study contributes to this line of work by providing a resource-rational process-level explanation of two (purportedly irrational) effects in risky decision-making. As such, our work suggests an alternative perspective on evaluating human rationality: To judge human rationality not by whether human behavior respects a set of axioms and/or principles (e.g., the independence axiom, the betweenness axiom, etc.), but by the extent to which human judgment and decision-making is in accord with rational process models acknowledging computational and cognitive limitations (e.g., Lieder & Griffiths, 2018; Nobandegani, 2018).

While the foregoing approach to evaluating human rationality is still in its infancy, and much work is needed to investigate the efficacy of this perspective, we hope to have made some progress in this direction.

Appendix

Proof of Theorem 1

There are three possible pairs $(T,C),(T,D),(C,D)$ that the decision-maker can begin with. In what follows we consider each possibility in turn, showing that only starting with the pair (C,D) grants the occurrence of risky DE.

Case 1. Starting with the pair (T,C):

$$\mathbb{P}(T|\{T,C,D\}) = \mathbb{P}(T|\{T,C\})\mathbb{P}(T|\{T,D\}) \leq \mathbb{P}(T|\{T,C\}).$$

The right-hand side inequality immediately follows from the basic axiom in probability that $\mathbb{P}(T|\{T,D\}) \leq 1$.

Case 2. Starting with the pair (T,D):

$$\mathbb{P}(T|\{T,C,D\}) = \mathbb{P}(T|\{T,D\})\mathbb{P}(T|\{T,C\}) \leq \mathbb{P}(T|\{T,C\}).$$

The right-hand side inequality immediately follows from the basic axiom in probability that $\mathbb{P}(T|\{T,D\}) \leq 1$.

Case 3. Starting with the pair (C,D):

$$\begin{aligned}\mathbb{P}(T|\{T,C,D\}) \quad &= \quad \mathbb{P}(C|\{C,D\})\mathbb{P}(T|\{T,C\}) + \\ &\quad\quad \mathbb{P}(D|\{C,D\})\mathbb{P}(T|\{T,D\}) \qquad (8)\end{aligned}$$

Under the plausible assumption that $\mathbb{P}(T|\{T,D\}) > \mathbb{P}(T|\{T,C\})$ (due to the fact that T dominates D), it follows that the right-hand side of Eq. 8 is greater than $\mathbb{P}(T|\{T,C\})$, hence granting the occurrence of risky DE.

This completes the proof of Theorem 1. ■

References

Arrow, K. J. (1963). *Social Choice and Individual Values*. Yale university press.

Battaglia, P. W., Hamrick, J. B., & Tenenbaum, J. B. (2013). Simulation as an engine of physical scene understanding. *Proceedings of the National Academy of Sciences*, *110*(45), 18327–18332.

Bernoulli, D. (1738/2011). Exposition of a new theory on the measurement of risk. In *The kelly capital growth investment criterion: Theory and practice* (pp. 11–24). World Scientific.

Bonawitz, E., Denison, S., Griffiths, T. L., & Gopnik, A. (2014). Probabilistic models, learning algorithms, and response variability: sampling in cognitive development. *Trends in Cognitive Sciences*, *18*(10), 497–500.

Camerer, C. F., & Ho, T.-H. (1994). Violations of the betweenness axiom and nonlinearity in probability. *Journal of Risk and Uncertainty*, *8*(2), 167–196.

Cary, M., & Reder, L. M. (2002). Metacognition in strategy selection. In *In: Metacognition: Process, function and use* (pp. 63–77). Springer.

da Silva Castanheira, K., Nobandegani, A. S., & Otto, A. R. (2019). Sample-based variant of expected utility explains effects of time pressure and individual differences in processing speed on risk preferences. In: *Proceedings of the* 41*st* *Annual Conference of the Cognitive Science Society.* Austin, TX: Cognitive Science Society.

Gershman, S. J., Horvitz, E. J., & Tenenbaum, J. B. (2015). Computational rationality: A converging paradigm for intelligence in brains, minds, and machines. *Science*, *349*(6245), 273–278.

Gershman, S. J., Vul, E., & Tenenbaum, J. B. (2012). Multistability and perceptual inference. *Neural Computation*, *24*(1), 1–24.

Geweke, J. (1989). Bayesian inference in econometric models using monte carlo integration. *Econometrica: Journal of the Econometric Society*, 1317–1339.

Griffiths, T. L., Lieder, F., & Goodman, N. D. (2015). Rational use of cognitive resources: Levels of analysis between the computational and the algorithmic. *Topics in Cognitive Science*, *7*(2), 217–229.

Griffiths, T. L., Vul, E., & Sanborn, A. N. (2012). Bridging levels of analysis for probabilistic models of cognition. *Current Directions in Psychological Science*, *21*(4), 263–268.

Hammersley, J., & Handscomb, D. (1964). *Monte carlo methods.* London: Methuen & Co Ltd.

Hershey, J. C., & Schoemaker, P. J. (1980). Prospect theory's reflection hypothesis: A critical examination. *Organizational Behavior and Human Performance*, *25*(3), 395–418.

Hertwig, R., & Pleskac, T. J. (2010). Decisions from experience: Why small samples? *Cognition*, *115*(2), 225–237.

Huber, J., Payne, J. W., & Puto, C. (1982). Adding asymmetrically dominated alternatives: Violations of regularity and the similarity hypothesis. *Journal of Consumer Research*, *9*(1), 90–98.

Kahneman, D., & Tversky, A. (1979). Prospect theory: An analysis of decision under risk. *Econometrica*, *47*(2), 263–291.

Lake, B. M., Ullman, T. D., Tenenbaum, J. B., & Gershman, S. J. (2017). Building machines that learn and think like people. *Behavioral and Brain Sciences*, *40*.

Lieder, F., & Griffiths, T. L. (2018). Resource-rational analysis: understanding human cognition as the optimal use of limited. Available on Researchgate.

Lieder, F., Griffiths, T. L., Huys, Q. J., & Goodman, N. D. (2018). Empirical evidence for resource-rational anchoring and adjustment. *Psychonomic Bulletin & Review*, *25*(2), 775–784.

Machina, M. J. (1987). Choice under uncertainty: Problems solved and unsolved. *Journal of Economic Perspectives*, *1*(1), 121–154.

Markowitz, H. (1952). The utility of wealth. *Journal of Political Economy*, *60*(2), 151–158.

Maule, A. J., & Svenson, O. (1993). Theoretical and empirical approaches to behavioral decision making and their relation to time constraints. In *Time Pressure and Stress in Human Judgment and Decision Making* (pp. 3–25). Springer.

Mohr, P. N., Heekeren, H. R., & Rieskamp, J. (2017). Attraction effect in risky choice can be explained by subjective distance between choice alternatives. *Scientific Reports*, *7*(1), 8942.

Nobandegani, A. S. (2018). The Minimalist Mind: On Mininality in Learning, Reasoning, Action, & Imagination. McGill University, PhD Dissertation.

Nobandegani, A. S., da Silva Castanheira, K., O'Donnell, T. J., & Shultz, T. R. (2019). On robustness: An undervalued dimension of human rationality. In *Proceedings of the* 17*th* *International Conference on Cognitive Modeling.* Montreal, QC.

Nobandegani, A. S., da Silva Castanheira, K., Otto, A. R., & Shultz, T. R. (2018). Over-representation of extreme events in decision-making: A rational metacognitive account. In: *Proceedings of the* 40*th* *Annual Conference of the Cognitive Science Society* (pp. 2391-2396). Austin, TX: Cognitive Science Society.

Nobandegani, A. S., da Silva Castanheira, K., Otto, A. R., & Shultz, T. R. (2019a). A resource-rational process-level account the St. Petersburg paradox. In *Proceedings of the* 41*st* *Annual Conference of the Cognitive Science Society.* Austin, TX: Cognitive Science Society.

Nobandegani, A. S., da Silva Castanheira, K., Otto, A. R., & Shultz, T. R. (2019b). A resource-rational mechanistic approach to one-shot non-cooperative games: The case of prisoner's dilemma. In: *Proceedings of the* 41*st* *Annual Conference of the Cognitive Science Society.* Austin, TX: Cognitive Science Society.

Noguchi, T., & Stewart, N. (2014). In the attraction, compromise, and similarity effects, alternatives are repeatedly compared in pairs on single dimensions. *Cognition*, *132*(1), 44–56.

Poor, H. V. (2013). *An Introduction to Signal Detection and Estimation*. Springer Science & Business Media.

Prelec, D. (1990). A "pseudo-endowment" effect, and its implications for some recent nonexpected utility models. *Journal of Risk and Uncertainty*, *3*(3), 247–259.

Roberts, M. J., & Erdos, G. (1993). Strategy selection and metacognition. *Educational Psychology*, *13*(3-4), 259–266.

Russo, J. E., & Rosen, L. D. (1975). An eye fixation analysis of multialternative choice. *Memory & Cognition*, *3*(3), 267–276.

Scholten, M., & Read, D. (2014). Prospect theory and the "forgotten" fourfold pattern of risk preferences. *Journal of Risk and Uncertainty*, *48*(1), 67–83.

Simonson, I. (1989). Choice based on reasons: The case of attraction and compromise effects. *Journal of Consumer Research*, *16*(2), 158–174.

Soltani, A., De Martino, B., & Camerer, C. (2012). A range-normalization model of context-dependent choice: a new model and evidence. *PLoS Computational Biology*, *8*(7), e1002607.

Speekenbrink, M., & Shanks, D. R. (2013). *Decision Making.* In *The Oxford Handbook of Cognitive Psychology.* Reisberg (Ed.). Oxford University Press.

Svenson, O. (1993). *Time Pressure and Stress in Human Judgment and Decision Making*. Springer Science & Business Media.

Tsetsos, K., Chater, N., & Usher, M. (2012). Salience driven value integration explains decision biases and preference reversal. *Proceedings of the National Academy of Sciences*, *109*(24), 9659–9664.

Tversky, A., & Kahneman, D. (1973). Availability: A heuristic for judging frequency and probability. *Cognitive Psychology*, *5*(2), 207–232.

Tversky, A., & Kahneman, D. (1992). Advances in prospect theory: Cumulative representation of uncertainty. *Journal of Risk and Uncertainty*, *5*(4), 297–323.

Von Neumann, J., & Morgenstern, O. (1947/2007). *Theory of games and economic behavior (commemorative edition)*. Princeton University Press.

Vul, E., Goodman, N., Griffiths, T. L., & Tenenbaum, J. B. (2014). One and done? optimal decisions from very few samples. *Cognitive Science*, *38*(4), 599–637.

Wedell, D. H. (1991). Distinguishing among models of contextually induced preference reversals. *Journal of Experimental Psychology: Learning, Memory, and Cognition*, *17*(4), 767.