# Conceptually Plausible Bayesian Inference in Interval Timing

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## Introduction

In a world that is uncertain and noisy, human perception makes use of optimization procedures to reduce the influence of moment-to-moment noise by incorporating statistical properties of previous experiences. This observation holds for the perception of many psychophysical quantities, ranging from light intensity to interval timing, the focus of the current study. These types of optimization procedures assume that when a specific interval needs to be reproduced, observers do not only take the current percept into account but also use their prior knowledge of previous similar incidents to form an internal estimate of the just perceived interval, yielding a central tendency effect (Hollingworth, 1910). That is, in a reproduction task in which different durations have to be reproduced, the shorter intervals will be overestimated and the longer durations underestimated yielding a regression towards the mean. A formal account of this phenomenon has only recently been proposed. In 2010, Jazayeri and Shadlen formulated a mathematical framework in which a Bayesian observer is assumed to combine the noisiness associated with time perception with a probability distribution representing the earlier observed durations. The actual reproduction is based on the posterior distribution, which consist of the integration of a Gaussian-distributed likelihood, representing the observed duration, with a uniform prior, representing the experimental history. Jazayeri and Shadlen demonstrated that the mean of the posterior distribution captures a number of important empirical phenomena, including the central tendency effect.

To account for individual differences in the magnitude of the central tendency effect, they assumed differences in the variability of the temporal percept, represented in the width of the likelihood (see https://vanrijn.shinyapps.io/MaassVanMaanenVanRijn2019/ for a simulation). Note that after a value has been sampled from the posterior distribution, Gaussian-shaped production noise is applied to map the posterior-based estimation to the actually reproduced duration. Similar Bayesian observer models have been shown to accurately reproduce human behavior in a number of timing tasks (see, e.g., Shi, Church, & Meck, 2013).

From a theoretical or conceptual perspective, however, one can question certain implementation decisions underlying this Bayesian Observer Model. Firstly, the prior with which the likelihood is convolved is assumed to be a uniform distribution precisely spanning the range of the presented durations. Even though this provides computational simplicity, its theoretical suitability can be questioned as the average of the resulting posterior distributions will, because of the central tendency, have a higher mass around the center of the distribution. Following the assumption that the prior is based on previous posteriors, the prior should reflect this bias towards the mean. This example of the central limit theorem would suggest a more Gaussian-like distributed prior which also naturally results from instance-based explanations of the role of memory processes in interval timing (for a review, see Van Rijn, 2016). Cicchini et al. (2012) addressed the issue of the uniform prior, and proposed to use a truncated normal distribution to represent the prior. Where Jazayeri and Shadlen (2010) focused on the width of the likelihoods, resembling clock precision, to account for the variability between participants in observed central tendency effects, Cicchini et al. (2012) argued that the prior might also differ on a per participant basis. To tear apart the contribution of the likelihood and prior, they estimated clock variability using a secondary task. With the likelihood fixed on a per participant basis, they demonstrated that the width of a truncated normal distribution varied over participants.

Even though a Gaussian-like distribution is theoretically more plausible than a uniform prior, its theoretical elegance is affected by the necessity to constrain its range to prevent it extending to negative values, nor does it match the heavier right tail observed in empirical data. In addition, the proposed symmetrical, Gaussian prior does not match the stronger central tendency bias for the longer compared to the shorter durations: As the mass of the average posteriors associated with the longer durations is more pulled towards the mean of all presented durations than the average of the posteriors associated with the shorter durations, a skewed Gaussian distribution would be theoretically more plausible.

A second theoretical challenge for these Bayesian observer models is that they incorporate two independent sources of noise, one associated with the perceptual phase ($w_m$), determining the width of the likelihood, and one associated with the reproduction of a duration which is based on the posterior ($w_p$). Whereas $w_m$ captures the perceptual noise associated with perceiving the onset and offset of the presented duration, as well as the clock noise associated with the actual timing of the interval, $w_p$ captures the perceptual noise for the onset of the reproduction phase, the clock noise, and the motor noise associated with the motor movement to mark the end of the reproduction phase (by a key press). Assuming perceptual noise to be smaller than motor noise, and clock noise to be the dominant source of noise (e.g., Taatgen, Van Rijn, & Anderson, 2007), $w_p$ should always be larger than $w_m$. Additionally, as clock noise can be assumed to be the largest source of variability in both $w_m$ and $w_p$, it follows to estimate $w_m$ and estimate $\Delta w_p$ that expresses the difference in noise between a perception and motor action (i.e., the reproduction noise, $w_p$, is defined as $w_m + \Delta w_p$). As both parameters were fit independently in Jazayeri and Shadlen's Bayesian Observer model, $w_p$ could be estimated at a smaller value than $w_m$ and no correlation between both parameters was instantiated. In contrast, no parameters were estimated in Cicchini's et al. (2012) model. Their model incorporated an estimate for $w_m$ based on each participant's performance on a secondary task, and $w_p$ was fixed for all participants at a value that fell within the range of values that were determined for $w_m$. Thus, this model did not adhere to the notion that $w_m$ should be larger than $w_p$, and it assumed that all sources of noise, including clock noise, were identical for all participants during reproduction.

Here we present Bayesian Observer models with different assumptions with respect to the source of the individual differences, by considering individual differences in clock noise and memory: We will independently estimate $w_m$ and $\Delta w_p$ assuming priors based on either a fixed uniform prior distribution, or normal and log-normal shaped prior distributions of which the variance will be estimated. To assess the goodness of fit of these models, we will estimate fit measures for 15 aged participants with the diagnosis of amnestic Mild-Cognitive Impairment (aMCI) and 44 healthy aged controls. Whereas the first group showed strong central tendency effects, the latter group showed weaker effects (Maaß, Riemer, Wolbers, & Van Rijn, submitted). will be compared. Interestingly, measures of memory functioning predicted the magnitude of the central tendency effect, even in the healthy aged control group. Additionally, we will use 1-second task data (Maaß & Van Rijn, 2018) to assess clock variability. The results suggested that neither age (cf. Paraskevoudi, Balcı, & Vatakis, 2018), nor clinical status (cf. Rueda & Schmitter-Edgecomb, 2009) influenced clock time variability, but that aMCI patients more strongly weigh prior experiences than healthy, age-matched controls, resulting in stronger central tendency effects. By fitting Bayesian Observer models to the empirical data from these (sub)populations, we aim to understand the contributions of likelihood and prior on temporal reproduction in healthy and memory-impaired individuals.

In sum, we will (1) assess whether one type of prior is preferred, (2) whether estimated values that mostly reflect clock noise (i.e., $w_m$) correlates to the collected clock-variability measures, and (3) whether the estimated prior parameters provide a sensible theoretical interpretation of the empirical phenomena.

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