

Memory of relative magnitude judgments informs absolute identification

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Abstract

We characterize difficulties with both absolute and relative accounts of magnitude representation in the absolute identification paradigm and present a resolution for these difficulties. We postulate that people store neither long-term internal referents for stimuli nor operate simply using binary comparisons of size between successive stimuli. Rather, they obtain probabilistic judgments of size differences between successive stimuli and encode these for future use, within the course of identification trials. We set up a Bayesian ideal observer model for the absolute identification task using this memory-based representation of magnitude and propose a memory-sampling algorithm for solving it. Simulations suggest that this model captures complex human behavior patterns in absolute identification. Specifically, it reproduces empirically documented crossover effects, practice effects, effects from the use of overlapping stimuli and stimuli with uneven spacing.

Keywords: absolute identification; relative judgment; mental representations; memory; Bayesian learning

Introduction

While it is possible for humans to make fine-grained perceptual judgments about magnitudes, it is not yet clear at what granularity judgments about magnitudes experienced previously are stored. Theoretical opinion currently lies on a spectrum conceptually defined by two strongly divergent positions: one camp assumes that people have direct psychophysical access to the magnitude of entities in the world (how big was this stimulus on the scale I'm interested in?) (Brown, Marley, Donkin, & Heathcote, 2008); the other claims that people store only the relative results of comparative evaluations (which stimulus was bigger?) (Stewart, Brown, & Chater, 2005).

A classic problem for the absolute magnitude camp is that of absolute identification. Across a range of sensory modalities like line lengths, sound frequency, and sound loudness, observers are quicker and more accurate in identifying stimuli at the extremes of the presented stimulus set than those in the middle (Lacouture & Marley, 2004). In addition to identification, it is also possible to ask participants to categorize perceptual stimuli into one of two groups, in which case a similar pattern of results is seen to hold. The same 'bow-tie' seen in identification experiments is also seen in perceptual categorization experiments, with extreme stimuli within the stimulus set categorized more accurately and rapidly (Lacouture & Marley, 2004; Ratcliff & Rouder, 1998). Why should the range of stimuli presented in a set affect observers' responses to individual stimuli, if each stimulus has its own independent internal magnitude representation?

A classic challenge posed to the relative magnitude camp is the distance effect seen in closely related experiments. Distance effects are seen when people are asked to identify which of two presented stimuli is larger (Ratcliff & Rouder, 1998).

Participants were more accurate and quicker to respond when pairs of presented stimuli were far apart in actual brightness in a brightness discrimination experiment (Ratcliff & Rouder, 1998). The distance effect in perceptual choice finds an exact counterpart in the distance effect observed in economic experiments, where participants are more inconsistent and late in responding when the values of competing options are close (Dickhaut, Smith, Xin, & Rustichini, 2013). If people are not storing absolute magnitude information, why do they find stimuli farther apart easier to categorize and differentiate?

While explanations for subsets of these phenomena have been previously proposed, the ubiquity of these effects in perception and cognition demands a universal explanation, one equally applicable to both simple perceptual identification and cognitive preference judgment tasks. Sophisticated models of absolute identification place the source of these effects in the process by which observers map their internal representations of perceived stimuli magnitude onto discrete number labels. For instance, Lacouture & Marley showed that treating the magnitude-label mapping problem as an *encoder problem*, to be solved by a feed-forward network, yields mappings for response strengths quadratic in the stimulus order, immediately yielding the bowtie effect (Lacouture & Marley, 1995) when coupled with a DDM (Brown et al., 2008).

Assuming that long-term absolute internal representations of stimuli magnitude are noisy, the efficient encoding hypothesis holds that when confronted with a specific stimulus set, humans will respond to the specific task challenge of mapping stimuli to labels by comparing the presented stimulus to all available internal referents. The strength of the evidence for the mapping is information-theoretically stronger for stimuli corresponding to fewer overlapping internal referents, thus privileging points closer to the extremes, since they will have less interference from stimuli representations from one side of the scale.

A prominent empirical challenge to such accounts comes from the finding that stimuli of the same length are responded to differently when they are members of stimuli sets of different lengths, even within the same subject. If long-term stimulus magnitude representations exist, then they should be indifferent to the impact of adding more stimuli to an existing stimuli set, and the pattern of response should not change for the side of the stimulus order where new stimuli are not added. However, empirical evidence shows that it does (Sims, 2016). One solution to this problem is to adjust the noise levels in the internal stimuli representations 'adaptively' as a function of the set of stimuli to be represented (Sims, 2016). Such solutions, while mathematically feasible, call into question exactly how long-term the internal representations are, if

they are to be so responsive to extraneous context.

Adopting a representation of stimuli that stores only local comparisons, it has been argued that observers, once given feedback about the previous trial, and comparing the current stimulus to the immediately previous one, can restrict the range of possible responses by using the previous stimulus as an upper or lower bound for the new one (Stewart et al., 2005). This range restriction naturally proves to be more informative for stimuli closer to the edge of the stimulus set range, making responses to these stimuli more accurate. Thus, a convex relationship between response strength and stimulus order, specific to the presented stimuli set, is obtained.

Prominent challenges to such relative comparison-based accounts include the fact that they do not provide easy explanations for differences in response patterns induced as a function of unequal distances between stimuli in absolute identification tasks. When a large gap was included in the middle of an otherwise linear in log space stimuli range, people find stimuli surrounding this gap easier to identify. However, relative judgment models find it hard to even fit such data without detracting from predictive performance for the other stimuli (Brown, Marley, Dodds, & Heathcote, 2009). The core problem is that the model in question, the relative judgment model (RJM) uses a hard threshold in inter-stimulus distance to determine if a stimulus is larger or smaller than its predecessor, and fits this threshold as a parameter (Stewart et al., 2005). Changes in spacing end up compromising the quality of the model fit.

It is intriguing to note that what is hard to explain using one family of models is easy using the other. Relative judgment models would have no difficulty explaining the effect of multiple stimuli sets on the response pattern, since there are no long-term response strength mappings to expect consistent responses from. Absolute magnitude models would find it straightforward to explain heightened accuracy across large gaps - assuming the same variance for each internal representation, shifts in the mean by adding a gap increases the discriminability of neighboring stimuli, increasing the response strength for the corresponding stimuli.

Finally, both classes of models find it hard to explain practice effects in absolute identification - the fact that participants in these experiments actually get better at the task given practice (Rouder, Morey, Cowan, & Pealtz, 2004). Since neither class of model posits any form of learning mechanism for observers, they fail to explain the actual learning curves seen in real experimental subjects (Dodds, Donkin, Brown, & Heathcote, 2011).

Judgments are formed from memories

The striking complementarity of the strengths and weaknesses of absolute and relative models of absolute identification suggest an opportunity to formulate an intermediate account that bridges this theoretical divide. We make an effort to do so in this paper.

We make three assumptions about the process by which observers perform absolute identification and related tasks.

- First, we assume that the mental representation actually used by people in such tasks is a judgment of *relative* magnitude made using comparison to the immediately preceding stimulus during the experiment.
- Second, we assume that observers learn the stimulus-label mapping via a process well-described as an approximately Bayesian learning algorithm that explicitly samples memory engrams corresponding to the internal representations of stimulus magnitude learned during earlier trials of the experiment.
- Finally, we assume that this memory sampling self-terminates according to an information-gain criterion during each trial, and that the learned distribution of stimuli ranks at the time of termination is what the observer uses to emit an overt label response.

The relative magnitude representation. We use the same relative judgment assumption as Stewart’s RJM model (Stewart et al., 2005), that observers calculate a relative magnitude judgment comparing the immediate stimulus to the one immediately preceding it. This probabilistic representation of the pairwise difference between successive stimuli may, in principle, contain more information than a simple binary judgment. For any pair of successive observations $\{x_{t-1}, x_t\}$, we denote this probabilistic container of relative magnitude $p(r|x, o = \{x_{t-1}, x_t\})$, where r takes on the interpretation of magnitude. For all the demonstrations in this paper, we use binary judgments.

Bayesian stimulus-label mapping. Given this assumption about the nature of the long-term internal referent, an observer’s goal in absolute identification is to extract a relative magnitude judgment across stimuli in the stimulus set given access to a history of pairwise relative magnitude observations, and to do so using their own history of stimulus exposure within the task. We model the stimulus-label mapping process in the absolute identification task as Bayesian marginalization over relative magnitude judgments seen in pairwise comparisons (Srivastava, Vul, & Schrater, 2014). The mathematical machinery of sequential Bayesian updating allows us to formalize this learning process sequentially on a trial-by-trial, instead of treating the stimulus-label mapping and experimental responding as separate events as is classically done.

The relative magnitude of each stimulus, as we describe above, takes on a probabilistic interpretation formally expressed as $p(r|x, o)$, where r is the relative magnitude judgment, x is the currently visible stimulus, and $o = \{x_{t-1}, x_t\}$ is the relevant comparison *observation*. The ideal Bayesian observer learns $p(r|x, o)$ by combining comparison information from all previously observed comparisons. Thus, this quantity is obtained by marginalizing over the set of previously seen unique observations in memory $\mathcal{C} = \mathcal{P}(\mathcal{X}), s.t. \forall c \in$

$C, |c| = 2$ which we denote the memorized *comparisons*. Then,

$$D(x) = p(r|x, o) = \frac{\sum_c^C p(r|x, c) p(x|c) p(c|o)}{\sum_c^C p(x|c) p(c|o)}, \quad (1)$$

where it is understood that the *comparison* probability $p(c|o) = p(c|\{o_1, o_2, \dots, o_{t-1}\})$ is a distribution on the set of all comparisons available from observation history. Here, $p(r|x, c)$ encodes the probability that the item x was found to be larger in the comparison c , $p(x|c)$ encodes the probability that the item x was present in the context c and $p(c)$ encodes the frequency with which the observer encounters these comparisons during the experiment. This frequency is updated via recursive Bayesian estimation,

$$p(c^{(t)}|o^{(1:t)}) = \frac{p(o^{(t)}|c) p(c|o^{(1:t-1)})}{\sum_c^C p(o^{(t)}|c) p(c|o^{(1:t-1)})}. \quad (2)$$

This completes the computational description of the task an ideal Bayesian observer would perform in service of absolute identification, given access to local relative magnitude judgments. The practical approximation arises when we explicitly model the act of accessing previous relative magnitude judgments as memory sampling.

Self-terminating memory sampling. Evidence accumulation influences the shape of the distribution $p(c|o)$ via memory sampling. We model the process of memory recall as the activation of a subset Q of decision-relevant memory engrams. Using this notation, a general memory accumulation model could be expressed as,

$$p(c) = \sum_{q \in Q} p(c|q) p(q), \quad (3)$$

where $c \in C$ are stimuli comparisons available in memory and $q \in Q$ are memory engrams corresponding to past relative magnitude judgments. Here, the probability distribution $p(q)$ - which we call the *memory prior* - encodes the likelihood of recalling the memory of experience q , while the distribution $p(c|q)$ encodes the knowledge of having seen c and its corresponding relative magnitude judgment stored in the memory engram q . For simplicity, we assume a trivial bijective mapping between c and q - each memory engram is assumed to be associated with a unique stimuli pair.

This memory-sampling variant of $p(c|o)$ plugs directly as the prior in the Bayesian comparison probability update for $p(c|o)$ in Equation 2, which then itself plugs into the two computations in Equations 1 and 2 that define the ideal observer model. This replacement is facilitated by one additional assumption: that the comparison-specific memories recalled are episodic, and therefore convey all comparison-relevant information once the comparison episode itself has been activated in memory¹.

¹This assumption simplifies our analysis by ignoring the memory dependence of our other intermediate probability terms. While it is likely that such dependence exists, its effects will work in the same direction as the basic results of our approach, since it would further impoverish the preference representation we are already imposing sampling constraints on.

Finally, we formalize our information-theoretic criterion for terminating memory sampling and emitting an identification response. We assume that observers continue to sample memory engrams until the rate at which these provide new information subsides below a threshold. Additional information gained by adding an additional engram q_n to the existing set can be expressed as,

$$IG(q_n) = \sum_i p(c_i|q_{1:n-1}) \log \frac{p(c_i|q_{1:n})}{p(c_i|q_{1:n-1})}, \quad (4)$$

so our sampling termination rule is,

$$\arg \min_n IG(q_n) < T, \quad (5)$$

where T is the termination threshold, potentially informed by exogenous influences.

Stimulus rank decoding. At each time step t the model uses the differential internal representation between the current and one-back stimulus ($D_t - D_{t-1}$), and the previous rank obtained from post-trial feedback to estimate the current stimulus' rank according to the formula:

$$RANK_t = RANK_{t-1} + \left(\frac{D_t - D_{t-1}}{\alpha_t} \right) \quad (6)$$

The parameter α in turn is updated at each time step t as:

$$\alpha_t = \frac{1}{t} \sum_r \left[\frac{D_t - D_{t-1}}{RANK_t - RANK_{t-1}} \right] \quad (7)$$

The observer's choice is determined from the relative magnitude judgments across all x available at the time memory sampling is terminated. We count instances where the observer's decision variable predicts the correct rank of the stimulus introduced on individual trials as accurate responses. Samples to termination are directly interpreted as linearly scaled response times. Notice that the parameter α controlling the rank-magnitude mapping relies entirely on local one-back comparisons between magnitude judgments D , as in the RJM (Stewart et al., 2005).

Simulation Results

Modelling our *in silico* experiment design after the design reported in (Lacouture & Marley, 2004), we showed 20 instances of the model 40 copies each of N stimuli, asking them to assign number labels $1 \dots N$ to them. On each trial, agents updated their estimates for $p(r|x_t, o = \{x_t, x_{t-1}\})$ following the model described above. Since we assumed equal spacing on a log scale for stimuli as in the original experiment, we kept the relative magnitude judgments as 1 for simplicity, and used a threshold value $T = 10e - 7$ across all our experiments unless specified otherwise.

Our model reproduces the absolute identification results of (Lacouture & Marley, 2004), which are the baseline benchmark for absolute identification models (Brown & Heathcote, 2008). Accuracy exhibits a convex relationship with stimulus

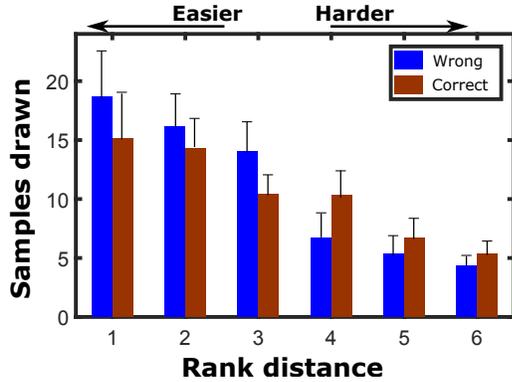


Figure 1: Replication of the crossover effect in perceptual choice. The x-axis plots the rank distance between compared stimuli on a given trial, and the y-axis plots the average number of samples drawn before responding during 20 model runs. Error bars represent ± 1 s.e.m. across these model runs.

order, and the response time distribution is concave, matching the profiles observed by (Lacouture & Marley, 2004). We do not explicitly demonstrate these results in the interest of succinctness, focusing on demonstrating more complex patterns of behavior.

Then, we show how it can replicate a harder pattern of behavior - the crossover effect in RT (Luce, Nosofsky, Green, & Smith, 1982; Brown & Heathcote, 2008).

Reproducing the crossover effect

The crossover effect describes a complicated pattern of behavior typically seen in perceptual choice experiments. When choice is easy and speed is emphasized, incorrect responses are quicker than correct responses; when choices are harder and accuracy is emphasized, the opposite is true (Brown & Heathcote, 2008). This pattern of response time (RT) behavior has proved very challenging for several models of choice RT to fit, and is a challenging benchmark for models in this field.

Perceptual choice fits into our framework without affecting the formalism in the slightest. The only difference is that the observations o now represent two stimuli seen together instead of sequentially. All the other interpretations remain identical to those in the identification setting. We conducted *in silico* experiments using the same simulation setup as above. As Figure 1 illustrates, our model displays a crossover effect even ignoring the effect of the speed-accuracy trade-off.

Further, our model offers a straightforward parameter-free explanation for the crossover effect. Simple choices correspond to situations where most samples in memory point in the same direction for a particular stimulus. In such cases, the only way the model could fail to produce the correct response is if the sampling was terminated prematurely. Thus, incorrect responses for simple choices have to be fast. Given

sufficient time for integration, it would be impossible for the model to be incorrect. Hard choices correspond to situations where both options have memory samples supporting their case for being bigger. In such cases, the model is biased towards terminating when the marginal information gain is low. Thus, the model will fail to terminate when memory sampling fails to resolve to a modal response, which is more likely when the sampling has failed to discover the true mode of the relative magnitude judgment distribution, resulting in bigger response times for errors.

Reproducing practice effects

By varying the number of history samples, i.e. the samples that the model is exposed to before the start of the trials, our model can reproduce the differential conditions observed in experiments documenting practice effects in absolute identification (Dodds et al., 2011). We ran 5 iterations of the model for practice/no practice conditions with number of stimuli $N = 6$. For the no practice case, the model was exposed to 30 history samples, whereas in the practice case it was exposed to 300 history samples before we started taking the model's predictions into account. The information threshold was kept at $T = 10E - 4$. The results of the simulation are shown in Figure 2 alongside data from (Dodds et al., 2011). A clear qualitative reproduction of the pattern of results seen in the experiment is seen - accuracy for end-points starts out high, while responses for stimuli in between start out with greater error, and then improve. The explanation is intuitive: fewer unique samples are needed to clearly differentiate the rank order of endpoint stimuli.

Reproducing overlapping stimuli effects

The overlapping stimuli effect in which the same stimulus elicits different responses when presented as part of different stimulus sets poses a challenge to absolute accounts of magnitude representation. We ran 30 iterations of the model, for each of the cases with number of stimuli $N = 5$, $N = 8$, and $N = 11$. To work around the large compute times necessitated by the combinatorial explosion in the number of contexts to be sampled with increase in N , the empirical data presented in (Sims, 2016) for the cases $N = 13$, $N = 20$, and $N = 30$ were down sampled to $N = 5$, $N = 8$, and $N = 11$ respectively. The down-sampling was done by taking every 3rd data point and extrapolating the last point, if necessary. We observe a strong qualitative and quantitative reproduction of the empirical effect with a single parameter fit across all conditions in the experiment.

Reproducing the uneven spacing effect

When a large central gap is introduced into the stimuli set, the accuracy profile significantly deviates from the bowtie curve, with the stimuli near the gap having higher accuracy compared to the ones away from the gap. The uneven spacing effect presents a major challenge to relative accounts of magnitude representation, including ours. To capture this effect, our model requires an additional augmentation - we assume

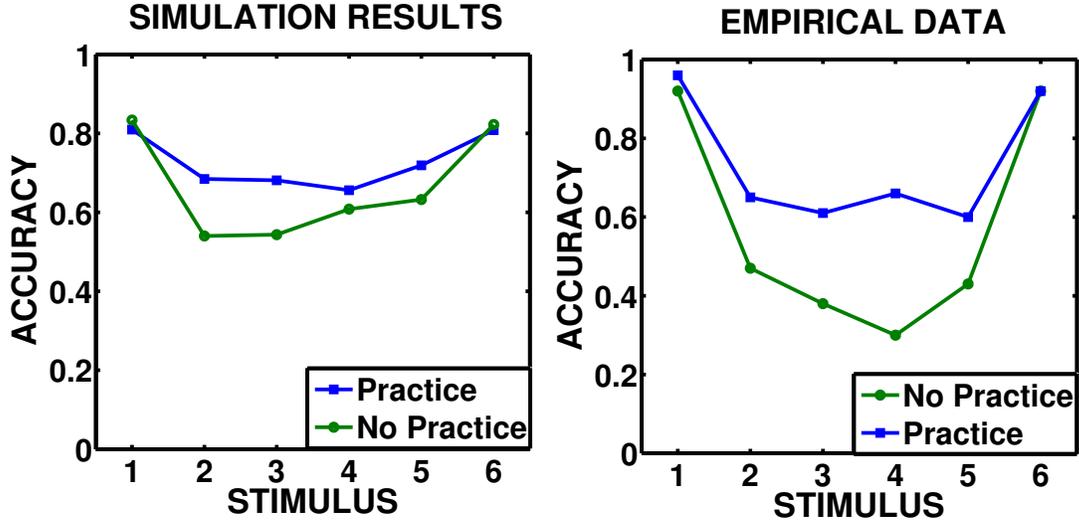


Figure 2: Demonstrating practice effects in absolute identification. (Left) Simulation results run by varying the number of history samples to which the model is initially exposed. (Right) Empirical data re-plotted from data in article (Dodds et al., 2011).

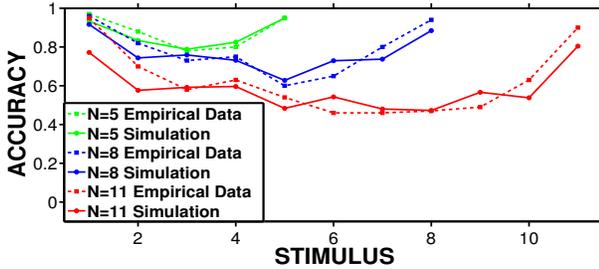


Figure 3: Reproducing the overlapping stimuli effect. The x-axis plots the stimulus length and the y-axis plots the accuracy of the model. Empirical data re-plotted from (Sims, 2016)

that the observer first seeks to identify when a stimulus from one side of the gap is presented versus the other, and then tries to identify its rank.

Rank decoding across gaps. Our model’s estimate of α as a simple running average makes sense when it expects perceptually even spacing between successive stimuli. Where spacing is uneven, ranks estimated using such an estimate would be faulty. To accommodate the effect of uneven spacing, after each trial, the decoder in our model calculates and stores the ordinal difference between the perceived rank, as estimated using α , and the real rank, obtained from feedback post trial. This rank difference is denoted by RD and is updated at every trial involving a jump across the gap.

$$RD_t = (\text{PerceivedRank})_t - (\text{RealRank})_t \quad (8)$$

Rank prediction occurs as follows here

$$RANK_t = RANK_{t-1} + \left(\frac{D_t - D_{t-1}}{\alpha} \right) - RD_t, \quad (9)$$

such that the evenly spaced stimulus set decoding (as specified in Equation 6) arises as a special case of the unevenly spaced stimuli. If the gap tracks the constant inter-stimulus interval, RD goes to 0 in Equation 9, yielding Equation 6.

We ran 30 iterations of the model with number of stimuli $N=10$ with a large central gap, 9 times the size of the even spacing gap, introduced between stimuli 5 and 6. The threshold parameter was held at the same value as in the other demonstrations. The model’s results (Figure 4) match the ‘w’ shaped accuracy profile observed in the empirical data.

Discussion

In this paper, we have presented a model of absolute identification based on three basic principles: one, that observers store 1-back relative magnitude judgments in memory; two, that observers solve the computational problem implicit in absolute identification (stimulus-rank mapping) using an approximately Bayesian calculation that can be stylized as sampling engrams from memory; three, that this memory sampling procedure terminates using an information-gain criterion.

Our model’s capacity to identify absolute stimuli arises from differences in the informativeness of memory samples corresponding to various stimuli. Because the evidence from comparisons involving extreme stimuli consistently points the same way, the marginal information gain from sampling

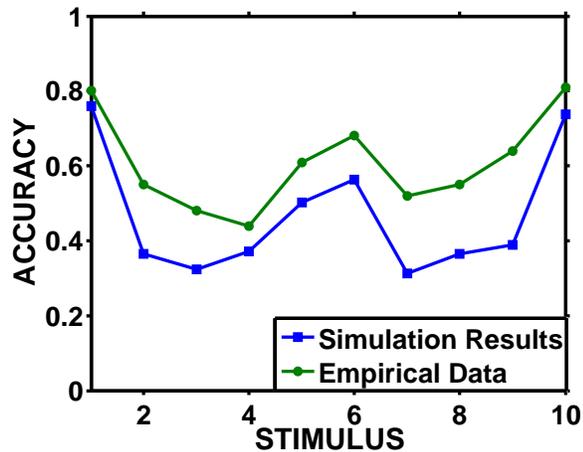


Figure 4: Reproducing the uneven spacing effect. The x-axis plots the stimulus length and the y-axis plots the accuracy of the model. Empirical data replotted from (Brown et al., 2009)

saturates rapidly and the model terminates memory retrieval sooner, leading to faster and accurate responses. On the other hand, for stimuli closer to the middle, samples will be split between comparisons where the stimulus is larger and ones where it is smaller, resulting in greater decision variable volatility, and hence, more sampling. This interaction then manifests summarily as slower and noisier responses.

The representational flexibility provided by our encoding stimulus-rank mapping information in memory, in conjunction with the fact that we model the process by which the representation is actually learned trial-by-trial from stimulus observations, allows our model to reproduce practice effects in absolute identification (Dodds et al., 2011), as well as reproduce both the shift in response patterns as a function of stimulus set (Rouder et al., 2004) and the heightened response to unequal spacing (Brown et al., 2009) without committing to long-term storage of magnitude estimates for arbitrary stimuli, and using only two explicit free parameters. Our account also predicts that the uneven spacing effect should also propagate to the RT distribution - an easily testable prediction.

In addition to these direct results, sequentially modelling the mapping process, in conjunction with the use of an information-based stopping criterion, also sheds new light on the relationship between the psychophysical bowtie effect (Lacouture & Marley, 2004) and the economic distance effect (Dickhaut et al., 2013). Extreme choice valence (distance in utility) appears to be correlated with lower error rate, response times and interestingly, levels of neuronal activation as measured by fMRI (Dickhaut et al., 2013). According to our model, constructing a decision variable using conflicting evidence requires more samples to breach the information-based threshold, resulting in greater effort, which is correlated with higher RT and brain activation for both perceptual and economic choices with greater mutual confusability, as

determined by their history of pairwise comparisons.

In summary, the model we have proposed appears to have robust empirical fits to challenging data within the absolute identification literature, and interesting theoretical connections with other strands in the literature on choice process modeling.

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