

# Degenerate Optimal Boundaries for Multiple-Alternative Decision Making

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## Abstract

Integration-to-threshold models of two-choice perceptual decision making have guided our understanding of the behaviour and neural processing of humans and animals for decades. Although such models seem to extend naturally to multiple-choice decision making, consensus on a normative framework has yet to emerge, and hence the implications of threshold characteristics for multiple choices have only been partially explored. Here we consider sequential Bayesian inference as the basis for a normative framework together with a conceptualisation of decision making as a particle diffusing in  $n$ -dimensions.

This framework implies highly choice-interdependent decision thresholds, where boundaries are a function of all choice-beliefs. We show that in general the optimal decision boundaries comprise a degenerate set of complex structures and speed-accuracy tradeoffs, contrary to current 2-choice results. Such boundaries support both stationary and collapsing thresholds as optimal strategies for decision-making, both of which result from stationary complex boundary representations.

This casts new light on the interpretation of urgency signals reported in neural recordings of decision making tasks, implying that they may originate from a more complex decision rule, and that the signal as a distinct phenomenon may be misleading as to the true mechanism. Our findings point towards a much-needed normative theory of multiple-choice decision making, provide a characterisation of optimal decision thresholds under this framework, and inform the debate between stationary and dynamic decision boundaries for optimal decision making.