

Timing and structure of Reward information influences bias in perceptual decisions as revealed by a hierarchical drift diffusion model

Manisha Chawla, Krishna P. Miyapuram

Centre for Cognitive and Brain Sciences, Indian Institute of Technology Gandhinagar,
Palaj, Gandhinagar 382 355, INDIA

kprasad@iitgn.ac.in

Abstract

Differential payoffs can bias simple perceptual decisions. Drift Diffusion models (DDM) have been successfully used to simultaneously model for response times (RTs) and accuracy of binary decisions. The DDM allows for identification of latent parameters that represent psychological processes underlying perceptual decisions. These parameters characterize decision making as a noisy process that accumulates evidence towards one of the two boundaries. Previous research in two alternative forced choice (2AFC) experiments has found that asymmetric payoffs result in a bias towards those decisions that result in higher payoff. We manipulate the reward structure resulting in symmetric and asymmetric payoffs for a simple orientation discrimination task and test for the differences in parameters of drift diffusion model that might relate to reward-induced bias in perceptual decisions. To understand the mechanisms of how reward information might be integrated with perceptual decisions, we altered the relative timing i.e. processing order of reward information and perceptual stimuli. Computational modelling using a hierarchical DDM revealed starting point bias towards stimuli oriented in the direction of higher rewards in asymmetric as well as symmetric rewards. The drift rate reflected the average reward expectation when reward information was presented before, but not after the perceptual stimulus. Our results suggest that integration of rewards with perceptual decisions is mediated by modulating motivation for evidence accumulation over time and prior bias in starting point.

Introduction

Computational models for Perceptual decision making describe the dynamic evolution of preferences across time until a decision is reached, rather than assuming a fixed state of preference. The Decision field theory (Bussemeyer & Townsend, 1993) is a member of a general class of sequential sampling models. Models such as drift-diffusion model (DDM: Ratcliff, 1978) suggest accumulation of varying sensory evidence that leads to a choice beyond a certain threshold. The DDM models decision-making in two-choice tasks represented by two boundaries separated by distance (represented by threshold parameter a). Lower threshold makes responding faster in general but increases the influence of noise on decision making and can hence lead to errors or impulsive choice. Higher threshold leads to more cautious responding (slower, more skewed RT distributions, but more accurate). Different studies have shown that the parameter a is sensitive to speed versus accuracy instructions (e.g., Voss et al., 2004). Additionally, there is a large body of research showing that age-related slowing in response time tasks can be partially explained by more emphasis on correct responses (e.g., Ratcliff et al., 2000, 2006, 2010, 2011). A drift-process accumulates

evidence over time with certain speed (drift-rate parameter v) until it crosses one of the two boundaries indicating the choice made. Due to noise in each trial of the drift process, the time taken to reach a particular boundary would vary across trials. If such a consistent variation is observed over different conditions the drift rate reflects task-difficulty with smaller drift rates representing more difficult tasks. In the comparison of participants, drift is a measure for individual cognitive or perceptual speed of information processing (Schmiedek et al., 2007). The DDM model also includes bias parameter z to account for starting point closer to one of the boundaries and non-decision time parameter t that encodes processes unrelated to decision making such as stimulus perception and movement (Smith & Ratcliff, 2004).

Previous research using the DDM has revealed that the effects of payoff manipulations on a perceptual decision-making task can be identified through various parameters. Dunovan et al. (2014) made a distinction between a prior bias in starting point parameter z and a dynamic bias in drift rate parameter v . The former model suggests influence of the payoffs on perceptual decisions to be only during the initial stage, while the latter suggests these influences to persist until reach the decision boundary. Van Ravenzwaaij et al. (2012) found that prior information influences starting point rather than the drift rate. Bias parameter is responsible for the starting point of response time distributions for each trial. Difference in bias parameter across conditions can reflect choices encountered with different payoff matrices. For example, Voss et al. (2004) showed that the starting point is moved toward a response threshold when the corresponding response leads to greater rewards (for a review see Voss et al., 2013). Similarly, in the domain of motivated perception, it has been found that the starting point is closer to the "positive" threshold than to the "negative" threshold in an evaluation task, even when expectancy values for both responses were symmetric (Voss et al. 2008). Diederich and Bussemeyer (2006) tested three models (1) the Bound Change Model that results in maximizing payoffs through a change in the decision threshold parameter, (2) the Drift - rate change model suggests a bias in drift rate owing to difference in payoffs, and (3) the Two-stage processing model proposed by Diederich (1997) where the decision task is separated as two evidence accumulation processes that occur sequentially. The first stage involves evidence accumulation process for the reward structure followed

by sensory evidence accumulation for the perceptual discrimination task. In a recent study, Diederich (2016) manipulated processing order of perceptual and payoff information and found further evidence in support of their multi-stage processing model. These prior studies establish a clear link for integration of contextual bias of reward information with perceptual decision making.

In the current study, we investigate how the temporal dynamics and structure of reward information bias perceptual decisions. The reward structure consists of two types of information - symmetric or asymmetric payoffs and is presented during an orientation discrimination task. As in previous studies, we presented reward information prior to the perceptual stimulus during the PreStim experiment. We conducted a second experiment, PostStim where the reward information was presented after the perceptual stimulus, but before requiring a response. Using hierarchical drift diffusion modeling, we tested separate models varying drift rate, decision threshold, and bias parameter to explain the choice distributions and response times. The models considered left and right responses as the two boundaries. Our results suggested that reward modulates perceptual decision making for both symmetric and asymmetric rewards encoded by a bias in starting point. Our results support the multistage model proposed by Diederich (2016) integrating the reward information in perceptual decisions.

Materials and Methods

Participants

PreStim experiment had 10 student volunteers (age range 19-31 years) and the PostStim experiment had 11 student volunteers (age range 19-31 years). All participants were right handed and had normal or corrected to normal vision. All participants gave written informed consent and were paid for their participation.

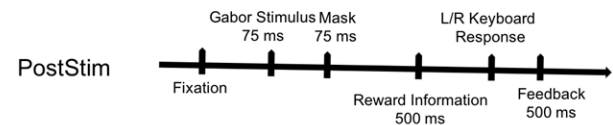
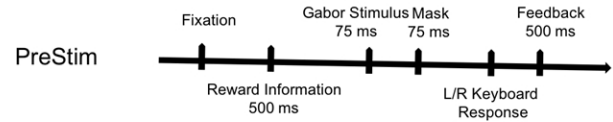
Stimuli

Each stimulus was composed of Gabor patches which were composed of a Gaussian envelope with a spatial frequency of 0.01 cycles/pixel. Maximal Michelson contrast of gratings was 0.9. Orientation of Gabor patches varied from -85 to 85 degrees with reference to vertical in step sizes of 5 degrees. A scrambled image was constructed by a combination of left and right oriented images of 45 degrees and was used for masking the gabor stimuli.

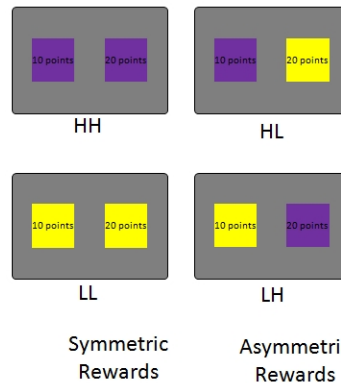
Design

The experiment was designed to test how reward information influences perceptual decisions. We manipulated the timing and the type of reward information presented that reflected the outcome (payoff) of the perceptual decisions. Therefore, reward information presented was irrelevant for performance of the perceptual task of detecting the orientation of gabor stimuli. Two types of reward information were presented - high reward magnitude (20 points) and low reward magnitude

A. Experiment Timeline



B. Reward Structure



C. Sample stimuli and Mask

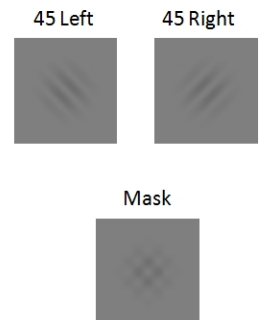


Figure 1: Experimental settings:- A. Structure of a single trial for both the PreStim and PostStim experiments. B. The four experimental conditions indicating High(H) and Low (L) rewards associated with left and right responses. Symmetric rewards are when both left and right responses would be associated with same reward. Asymmetric rewards conditions differentially reward left and right responses. C. Sample perceptual Gabor stimuli and mask stimulus. Participants were required to indicate the orientation of the Gabor stimulus.

(10 points). The reward information was displayed as the text written inside a square box and further coded in distinct colors. Reward information for the left and right oriented stimuli was presented on two sides of the screen centered vertically. The outcome of the trial was the reward displayed on the side that matched the orientation of the gabor stimulus. The reward information was manipulated in two ways - Symmetric and Asymmetric rewards. Symmetric conditions could be high rewards (HH) or low rewards (LL) in which both left and right correctly identified orientations were high or low rewarding, respectively. The Asymmetric conditions further was one of HL or LH conditions. In the HL condition, correct identification of left oriented gabor stimuli was rewarded with higher (20) points and the correct identification of right oriented gabor stimuli was rewarded with lower points (10). The LH condition was similar to the HL condition with the high and low reward contingencies being flipped.

Hierarchical drift diffusion modeling

We used an open source python toolbox for the hierarchical Bayesian estimation of the drift-diffusion model parameters (Wiecki et al., 2013). The toolbox uses Markov chain monte-carlo (MCMC) inference algorithm to estimate the joint posterior distributions of the different model parameters. We used Gelman-Rubin statistic to assess the convergence of the Markov chains by comparing the inter-chain and intra-chain variance of 5 different runs of the same model, resulting in ± 0.01 MC error suggesting 15000 samples were sufficient for convergence. For each model we generated 15000 posterior samples and discarded the first 5000 samples using burn to allow the MCMC chains to stabilize. The models were response coded with correct responses for right orientation terminating at the upper boundary and the left responses at the lower decision boundary.

To examine which model parameters are affected by the different type of reward structures and their timings we ran three different models allowing the parameters v , a and z to vary across experimental conditions, one at a time (model-V, model-A, model-Z), a composite model all three parameters to simultaneously vary between conditions, and a base model in which non of the parameters were allowed to vary across conditions. We then compared these different models using deviance information criterion (DIC) and posterior predictive checks (PPC) to find the best fitting model. DIC is a measure of relative goodness of fit for hierarchical Bayesian models (Speiegelhalter et al., 2012). DIC uses the trade-off between model fit and model complexity to compare relative goodness of the models. The best model is regarded as the one with the lowest DIC values. Difference of greater than 10 between different model DIC values is regarded as significant (Dunovan et al., 2014; Zhang & Rowe, 2014). Since, it is known that DIC is sometimes biased towards models with higher complexity we also ran Posterior Predictive Checks on group and subject data to assess the best fitting model (Michmizos & Krebs, 2014). We generated 500 simulated datasets from posterior predictive distributions of parameters corre-

sponding to the Composite model that was best-fit to the data based on lowest DIC value. We then compared the observed data distribution (empirical values from our experiment) with the simulated data generated which were found to be within 95% credible interval. Model goodness of fit is assessed using the mean standard error (MSE). Comparatively lower values of MSE for a model suggest that the model is able to reproduce observed data pattern distributions with less variability and more accuracy (Michmizos & Krebs, 2014).

The model parameters a , v and z thus estimated from the best fit model and their posterior distributions were used for statistical analysis. Our primary goal of the current research was to identify whether or not the different parameters varied across different conditions. We use posterior comparison for significance testing by calculating the proportion of overlap between the probability density of the two conditions being compared (Wiecki et al., 2013; Michmizos & Krebs, 2014). We also performed classical significance tests on mean parameter estimates as described further in Results section (Zhang & Rowe, 2014).

Results

Drift diffusion models

The drift diffusion models we considered were computed systematically allowing one parameter to vary across conditions keeping the other parameters invariant. Three models were formed to explore the modulations of parameters V , A , and Z : model-V, model-A, and model-Z, respectively. Model-V assessed for different drift-rates of evidence accumulation for left and right oriented perceptual stimuli across symmetric and asymmetric reward conditions. Similarly, model-A and model-Z assessed for decision threshold and starting point, respectively, for any biases in these parameters dependent on the reward structure. These three models were compared to a basic model in which all parameters remained invariant across conditions (BASE) using Deviance Information Criterion DIC. It was observed that all three models had lower DIC (model-V: 3426.32, model-A: 2950.80, model-Z: 3650.63) than BASE model indicating better fit to the data (3707.42). We ran a composite model that allowed for the above three parameters to vary across the four reward conditions that was found to be the best fit model (2782.35). The parameters estimated from the composite model were comparable to the independent models and future analysis are based on the estimates from the composite model. We ran a composite model including non-decision time (t) as a parameter. Mean estimates of parameter ' t ' were close to zero across reward conditions and hence are not discussed further.

Group parameter estimates from model fits of individual subjects were estimated using the hierarchical DDM. Group parameter estimates were tested for differences within the symmetric (HH and LL) and asymmetric (HL and LH) conditions using two complementary approaches (Zhang & Rowe, 2014). We compared mean parameter estimates across participants using a classical frequentist approach (i.e. t-test). We

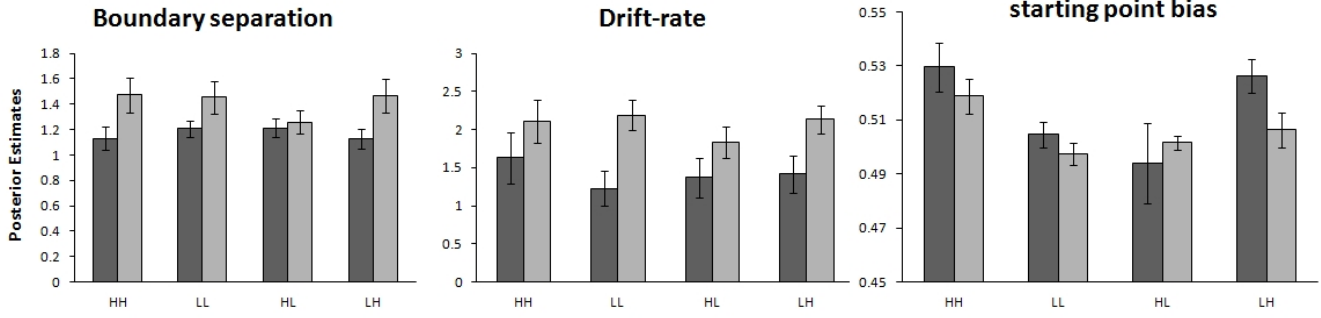


Figure 2: Group posterior estimates (y-axis) of the hierarchical drift-diffusion model parameters for the PreStim (dark gray bars) and PostStim (light gray bars). a.) Boundary separation parameter a b.) Drift-rate variability v and c.) Bias in starting point z . Error bars show standard error of mean from the posterior estimate samples.

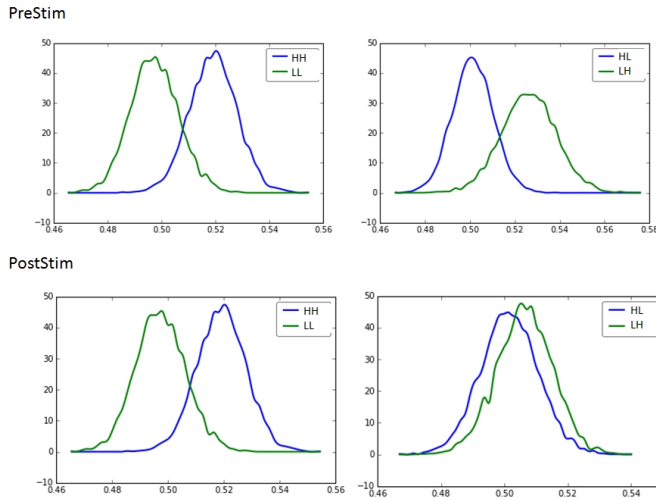


Figure 3: Posterior estimates (y-axis) of bias parameter z for a.) PreStim Symmetric conditions b.) PreStim Asymmetric conditions c.) PostStim Symmetric conditions d.) PostStim Asymmetric conditions.

compared the group posterior distributions obtained for each parameter using Bayesian approach.

Drift rate In PreStim experiment, we found bias towards high reward for drift rate corresponding to symmetric conditions ($HH > LL, t(9) = 2.47, p < 0.05; P_{Bayes} = 0.84$) but not for asymmetric conditions ($HL > LH, t(9) = -0.32, p = 0.38; P_{Bayes} = 0.46$). The drift rate for asymmetric conditions (HL, LH) was found to be intermediate to the symmetric high (HH) and low (LL) conditions possibly reflecting expectation of reward (Figure 2). In PostStim experiment we found no difference within the two symmetric ($HH > LL, t(10) = -0.17, p = 0.43; P_{Bayes} = 0.43$) and the two asymmetric reward ($HL > LH, t(10) = -1.24, p = 0.12; P_{Bayes} = 0.27$) conditions. Overall drift rates for the PostStim experiment were higher compared to the PreStim Experiment reflecting faster

response times (Figure 2). This could be due to the process of evidence accumulation being initiated during the reward information period prior to making a response.

Decision Threshold The decision threshold parameter showed no significant difference for both PreStim ($HH > LL, t(9) = -1.05, p = 0.16; P_{Bayes} = 0.18$) and ($HL > LH, t(9) = 1.37, p = 0.10; P_{Bayes} = 0.79$) and PostStim ($HH > LL, t(10) = 0.13, p = 0.45; P_{Bayes} = 0.51$ and ($HL > LH, t(10) = -1.38, p = 0.09; P_{Bayes} = 0.19$) experiments across conditions. This reflects that the boundary separation between left and right choices was not significantly different for the two symmetric and asymmetric reward conditions. The decision threshold for PostStim was observed to be greater than PreStim experiment (Figure 2) possibly due to the accumulated evidence not being allowed to reach the boundary (i.e. more conservative) while waiting for the "go" signal before the response execution.

Bias The posterior estimates of the response bias parameter for the PreStim experiment were found to be different in both the symmetric and asymmetric reward conditions. High reward condition showed relatively higher bias as compared to the low reward condition ($HH > LL, t(9) = 2.64, p < 0.05; P_{Bayes} = 0.93$). The asymmetric reward conditions had significant bias towards the boundary with high reward compared to the low reward ($HL > LH, t(9) = -2.08, p < 0.05; P_{Bayes} = 0.03$). These results reflect a prior bias for the starting point of the drift process towards the boundary with higher reward (Figure 6). For the PostStim experiment, the bias parameter was significantly different in symmetric reward conditions ($HH > LL, t(10) = 2.49, p < 0.05; P_{Bayes} = 0.96$), but was not significantly different for asymmetric rewards ($HL > LH, t(10) = -0.68, p = 0.25; P_{Bayes} = 0.36$). These results reflect absence of response bias when reward information is presented after the stimulus. On average, the bias parameters were similar for PreStim and PostStim experiments (Figure 3).

Discussion

Our results of bias parameter being dependent on reward structure supports the two-stage model proposed by Diederich and Busemeyer (2006). The timing of our experiments allows us to explicitly test support towards the two-stage model, specifically the mechanisms involved in integration of reward values during perceptual decisions. The model proposes two accumulation processes. Payoffs influence the starting point in the first stage by introducing a prior bias towards the response with higher reward. Then in the second stage, evidence accumulation is done for the perceptual stimulus. By manipulating the relative timing of presentation of reward information and the stimuli, we tested whether a dynamic bias can be induced by reward structure after the process of evidence accumulation has already begun upon stimulus presentation. Our results support the two-stage model as we find a bias in starting point for asymmetric rewards when the reward information is presented before but not after the stimulus presentation (Figure 3). The differences in starting point can therefore be attributed to the first stage of evidence accumulation process initiated by reward structure. When the reward information is presented after the stimulus, the two stage model would correspond to the second stage alone, in which evidence accumulation occurs for perceptual stimuli (Diederich & Busemeyer, 2006). Hence, we do not observe differences in starting point in PostStim experiment for asymmetric rewards.

Bias parameter encodes both symmetric and asymmetric rewards when reward information is presented before the stimulus. Further, when the reward information is presented after the stimulus, the bias parameter no longer encodes starting-point bias, rather encodes a decision bias for symmetric rewards. Similar proposal to distinguish between response-execution bias from decision bias have been made earlier (Voss et al., 2010). Reward conditions with high rewards (HH) have higher starting point relative to the reward conditions with low rewards (LL). This could possibly be due to greater motivation for perceptual discrimination in high reward conditions. This motivation-induced decision bias in decision needs to be interpreted differently from a response bias. Response bias refers to the starting point of evidence accumulation for the perceptual stimuli being biased towards the boundary corresponding to higher reward. This response bias was observed when the reward information was presented before, but not after the stimuli. The starting point in an unbiased setting would be midway of the two decision boundaries. Allowing the response bias to be estimated as a free parameter, which in turn allows us to re-interpret the bias parameter as a decision bias. Our results can be compared to previous findings (Voss et al. 2008) of motivational influences for perceptual and judgmental bias in which starting point parameter is biased towards the gain threshold. However, the current research does not explicitly dissociate the specific interpretation of the bias parameter arising from a response bias, or can be considered to be a decision bias.

The influence of reward structure on perceptual decisions can be described by two kinds of bias. The starting point reflects a prior bias, while the drift rate can encode for a dynamic bias (Dunovan et al., 2014). We found the drift rate encodes an average reward expectation for the PreStim Experiment. Our computational models estimate a single drift rate towards the two boundaries with complementary sign (v , $-v$). Hence, the finding that higher drift rate for high compared to low reward conditions (Figure 2) reflects a dynamic bias in processing the perceptual stimuli, consistent with previous research that claim that influence of payoffs persists over time, rather than only changing the starting-point (Dunovan et al., 2014; Voss et al., 2008). Together with other findings that do not find encoding of a dynamic bias in drift rate (e.g.: Mulder et al., 2012), and our manipulation of processing order, our results suggest that the reward information is encoded differently by prior bias in starting parameter and average reward expectation by dynamic bias i.e. drift rate parameter.

An intriguing result is that when the parameters from Post-Stim Experiment are compared to the PreStim experiment, we find higher drift rate and decision threshold (Figure 2). These could reflect the fact that the evidence accumulation process in support of the perceptual decision had already taken place before the reward information, following which the cue for indicating the response is given. Unlike previous studies that investigated influence of payoffs on subsequently presented perceptual stimuli, our study design separates the timing of choice execution from the perceptual decision process. Thus, we can dissociate whether the reward information influences the choice execution or the evidence accumulation mechanisms of the decision process. The results supports the notion that when stimulus is presented prior to the payoffs, evidence accumulation processes result in faster and more accurate responses, i.e. higher drift rate and more conservative decision thresholds.

Our study contributes towards understanding the mechanisms of integration of reward (value-based) information with perceptual decisions. Previous research by Rorie and colleagues (Rorie et al., 2010) demonstrated that perceptual decisions by monkeys being influenced by asymmetric but not symmetric rewards. Using computational model (DDM) analysis in the current research, we are able to identify latent parameters that correspond to influence of payoffs in perceptual decision making. The reward information pertains to value-based computations, but is unrelated to performance of the perceptual task. The reward information indicates the payoff (outcome) arising after perceptual decision. Our results demonstrated that parameters of the drift diffusion model, a model of perceptual decisions are influenced by the reward structure. This finding, though not completely novel, is further corroborated with manipulation of processing order (i.e. timing) to study the mechanisms of integration of reward values with perceptual decisions. While the behavioral results might simply suggest that it is crucial for reward information to be presented before, but not after the perceptual

stimulus, the computational modeling approach has been useful to understand the specific parameters that are encoded by the timing and structure of reward information on perceptual decisions.

In sum, our results show that symmetric and asymmetric rewards bias the starting point towards stimuli oriented in the direction of higher rewards, and also reflect the average reward expectation by the drift rate. These results can be interpreted as integration of rewards with perceptual decisions is mediated by modulating motivation for evidence accumulation over time and prior bias in starting point.

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