An ACT-R Model of Order Effects

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Abstract

Models based on classical probability theory have difficulty accounting for order effects, which occur when the order of question presentation affects response probabilities. Recently, quantum models have garnered support as an account of order effects. In particular, the pattern of order effects is consistent with a critical property of the quantum model called the QQ equality. We investigate whether the ACT-R cognitive architecture can produce order effects and satisfy the QQ equality based on memory retrieval mechanisms. In the ACT-R model, the answer to the first question creates a new context through which spreading activation influences retrieval probabilities for the second answer. Our analysis shows that spreading activation can produce order effects and satisfy the QQ equality, depending on the composition of declarative memory. Across a wide range of conditions, violations of the QQ equality are typically small, but moderate to large in a smaller set of cases.

Keywords: ACT-R; Quantum cognition; Order effects

Introduction

An order effect occurs when a response depends on the order in which stimuli are presented. In cognitive science, order effects are commonly treated as a nuisance factor in experimental design and data analysis. Typically, stimulus order is counter-balanced, marginalized out, and subsequently ignored. Recently, however, there has been growing interest in developing theoretical accounts of order effects (Trueblood & Busemeyer, 2011; Jones, Curran, Mozer, & Wilder, 2013; Wang, Solloway, Shiffrin, & Busemeyer, 2014). What makes order effects interesting is that they are difficult to account for using models based explicitly or implicitly on the foundation of classical probability theory. In particular, order effects violate the commutative law of classical probability theory according to which $\Pr(A \wedge B) = \Pr(B \wedge A)$. Furthermore, order effects are interesting because they highlight the context-dependent nature of cognition.

One example of order effects comes from a national poll asking respondents about the trustworthiness of former President Clinton and former Vice President Gore in two separate questions. One set of respondents judged "yes" or "no" whether Clinton is generally trustworthy followed by the same judgment about Gore, whereas the other set of respondents made the same judgments in the opposite order. In Table 1, the results for both question orders are transcribed from (Wang et al., 2014). The third sub-table shows the order effect, calculated as the difference between corresponding cells of the Gore-Clinton table and the Clinton-Gore table. A clear violation of the commutative law can be seen, as the values differ from zero.

During the 20th century, quantum probability theory was developed to account for order-dependence of measurement in physics. More recently, it has been adopted in cognitive science in response to similar violations of classical probability theory found in human cognition (Busemeyer, Pothos, Franco, & Trueblood, 2011; Atmanspacher, Römer, & Walach, 2002). Quantum probability theory is based on an alternative set of axioms which violate the commutative law under specific conditions. Several lines of research provide strong support for quantum probability account of order effects. For example, a quantum model provided a superior account of sequential belief updating compared to several competing models (Trueblood & Busemeyer, 2011). Perhaps the most compelling line of evidence is that the results from a large corpus of surveys and several experiments were consistent with a critical property of quantum probability theory called the Quantum Question (QQ) equality (Wang et al., 2014). The QQ equality is a structural property of the quantum model that constrains the possible patterns of order effects.

Very few alternative accounts of order effects have been proposed to date—perhaps reflecting the inherent challenge of the task. From a purely mathematical perspective, a Bayesian updating model can produce order effects with the inclusion of order-dependent events (Trueblood & Busemeyer, 2011). However, such a model is problematic because it is saturated, and thus merely re-describes any pattern of data without providing a principled way to assign values to its numerous terms. Recently, a multinomial processing tree called the repeat-choice model was developed to account for order effects and the QQ equality (Kellen, Singmann, & Batchelder, 2018). Multinomial processing trees use a tree-like structure of processing stages to describe choice behavior. According to the repeat-choice model, there is a probability distribution of over preference states, such as the preference to respond yes to both questions. With some probability, the responses will be made based only on the preference state. However, additional information will be considered with the complementary probability. If additional information is considered, there is some probability that the second response is the same as the first (assimilation effect) and the complimentary probability that the second response will differ from the first (contrast effect). Some variations of the repeat-choice model provided a similar fit to the data as the quantum model, thus demonstrating that model based on classical probability theory can account for the data.

One question that remains is whether a cognitive architecture can account for order effects. A cognitive architecture is a framework for simulating and developing unified theories of cognition (Newell, 1990). The primary goal of cognitive architectures is to provide a broad account of human cognition, spanning areas such as memory, multi-tasking, and perception among others. For this reason, order effects provide a new and important benchmark for testing cognitive architectures. Our goal in the present paper is to develop a model of order effects based on the ACT-R cognitive architecture (Anderson et al., 2004) and outline its predictions. Order effects present an interesting challenge for ACT-R because, unlike the quantum model—which was developed in physics to account for order dependence in measurement—it was not developed specifically to account for order effects. Instead, order effects must emerge from existing cognitive processes and mechanisms postulated by ACT-R. In what follows, we will demonstrate that ACT-R can produce order effects using memory retrieval mechanisms. In some cases, the model satisfies the QQ equality, while in other cases it violates the QQ-equality to varying degrees. The pattern of predictions depends critically upon the composition of declarative memory.

Overview

The remainder of the paper is organized as follows. We will begin with a brief overview of the quantum model and introduce the QQ equality. Next, we will provide a formal description of the ACT-R model of order effects. In the following section, we will describe its properties in terms of order effects and the QQ equality. Finally, we will discuss directions for future research.

Quantum Model

Quantum probability theory is based on an alternative set of axioms which allows non-commutative behavior (Busemeyer et al., 2011). Unlike classical probability theory in which events are subsets of a universal set, quantum probability theory is based on a geometric representation of uncertainty. Events are sub-spaces within an n dimensional vector space called a Hilbert space. A cognitive state represented as a state vector which is a linear combination of basis vectors that define the Hilbert space. In the quantum model, probabilities are formed by projecting the state vector onto a target subspace and computing the squared magnitude of the projection. A key distinction between classical and quantum probability theory is the concept of compatibility. Compatible events can be evaluated with respect to the same basis vectors, in which case quantum and classical probability theory make the same predictions. By contrast, incompatible events cannot be evaluated with respect to the same basis vectors. Instead, the basis vectors are rotated to create new set of basis vectors for the incompatible events. In other words, incompatible events cannot be evaluated simultaneously. Importantly, rotation leads to non-commutative behavior and other violations of classical probability theory. At a psychological level, rotation of the basis vectors represents a change of perspective.

Table 1: Joint probability table for Clinton-Gore order, the Gore-Clinton order, and the order effect. Column and row labels C and G correspond to Clinton and Gore. Subscripts y and n correspond to yes and no.

Clinton-Gore

	G_y	G_n
C_y	0.4899	0.0447
C_n	0.1767	0.2886

Gore-Clinton

	G_y	G_n
C_y	0.5625	0.0255
C_n	0.1991	0.213

Order Effect

	G_y	G_n
C_y	-0.0726	0.0192
C_n	-0.0224	0.0756

Although rotation of the vector space provides the quantum model with the flexibility to produce order effects, the range of behavior is highly constrained by a critical property known as the QQ equality (Wang et al., 2014). The QQ equality imposes a symmetrical relationship on the order effects in which both the diagonal elements and off-diagonal elements of the difference table must sum to zero. For example, in the third sub-table of Table 1, the diagonal elements $-.0726 + .0756 \approx 0$ and the off-diagonal elements $-.0224 + .0192 \approx 0$. Importantly, the QQ equality holds regardless of the initial state vector, the degree of rotation of the basis vectors, and is preserved in aggregated data (Wang et al., 2014). Formally, the QQ equality is defined by the following two statements:

$$q_1 = \Pr(Y_c = \text{yes} \wedge Y_g = \text{yes}) - \Pr(Y_g = \text{yes} \wedge Y_c = \text{yes}) + \Pr(Y_c = \text{no} \wedge Y_g = \text{no}) - \Pr(Y_g = \text{no} \wedge Y_c = \text{no}) = 0 \quad (1)$$

$$q_2 = \Pr(Y_c = \text{yes} \wedge Y_g = \text{no}) - \Pr(Y_g = \text{no} \wedge Y_c = \text{yes}) + \Pr(Y_c = \text{no} \wedge Y_g = \text{yes}) - \Pr(Y_g = \text{yes} \wedge Y_c = \text{no}) = 0 \tag{2}$$

where Y_p represents the response to question about person $p \in \{c, g\}$, and c denotes Clinton and g denotes Gore. Throughout, we will designate Y_p as a random variable and y_p as a specific realization of Y_p.

ACT-R Model

We developed a memory-based model of order effects within the ACT-R cognitive architecture (Anderson et al., 2004). ACT-R operates as a production system and is organized as a set of specialized processing modules which includes memory, visual/auditory perception and motor execution. Each module can process only one request at a given time and contains a buffer that holds a maximum of one chunk of declarative knowledge. For our present purposes, we will focus primarily on the declarative memory module. Although we will frame the model in terms of the Clinton-Gore example above, the model is applicable to many other cases in which responses are based on memory.

The model assumes that declarative memory contains chunks which represent true or false statements made by Clinton or Gore. When a question about a person is posed, a retrieval request is issued to declarative memory where the most active chunk about the target person is returned. The answer to the question is yes if the chunk contains a true statement. By contrast, the answer to the question is no if the retrieved chunk contains a false statement. During the retrieval of the first answer, there is no influence of contextual information. However, the answer to the first question creates a new context for answering the second question. In particular, the chunk for the first answer is stored in the imaginal buffer where activation spreads to chunks in declarative memory that share the same truth value, resulting in order effects under specific conditions.

Declarative Memory

Within ACT-R, a chunk is a basic unit of declarative knowledge given by a collection of slot-value pairs. For example a memory chunk could contain the slot 'name' with value 'Sigma' and the slot 'animal-type' with value 'dog'. For the remainder of the paper, we use the following notation for chunks: We use c_m to indicate a chunk in memory, where m is an index that ranges over all of the chunks in memory. We write the relationship between the slot s and value v for chunk m as $c_m(s) = v$. We will also need to reference the slots in the chunk for which a value is defined, which we denote Q_m. Note that all chunks do not necessarily (and generally do not) have the same slots, so to maintain the generality of the notation, we assume $c_m(s) = \emptyset$ for any s that is not a slot in chunk c_m i.e., for all $s \notin Q_m, c_m(s) = \emptyset$.

Like chunks, retrieval requests in the ACT-R architecture are collections of slot-value pairs, which we designate using set notation, $\mathbf{r} = \{(s_i, v_i)\}_{i \in I}$.

Knowledge Representation

Each chunk c_m in declarative memory contains a name slot, a statement slot, and a truth slot: $Q = \{\text{name}, \text{statement}, \text{truth}\}$. The name slot contains the name of the person, the statement slot contains the content of the statement and the truth slot contains the truth value of the statement.

Activation

The probability of retrieving a chunk increases monotonically with its activation value. Activation for chunk m is the sum of the following three components:

$$a_m = \beta + S_m + \varepsilon_m \tag{3}$$

where β is the base level constant, which scales activation up or down, S_m is spreading activation, and $\varepsilon_m \sim \text{Normal}(0, \sigma)$ is normally distributed noise. Spreading activation reflects the influence of context whereby active information in the architecture facilitates the retrieval of chunks containing the same values. Spreading activation has been used to explain the fan effect whereby concepts associated with more facts require more time to retrieve (Anderson, 1974). In the model, activation can only spread from the truth value of the chunk in the imaginal buffer to chunks in declarative memory. This follows from the simplifying assumption that statements are unique, and thus do not contribute to spreading activation. Given these simplifications, we can express spreading activation as:

$$S_m(x) = \frac{\gamma + \log\left(\frac{1}{1+x}\right)}{|Q|}$$

where x is the number of chunks in declarative memory with a same truth value as $c_{r,\text{imaginal}}(\text{truth})$, γ is the maximum association parameter, and $|Q|$ is the number of slots in each chunk, which is 3 in this case.

Retrieval Process

After the first question is encoded, a retrieval request $\mathbf{r} = \{(\text{name}, \text{v}_1)\}$ is submitted to declarative memory where a set of matching chunks $R = \{c_m \in M : (\text{name}, \text{c}_\text{m}(\text{name})) \in \mathbf{r}\}$ compete for retrieval and the chunk with maximum activation $c_r \in R$ is retrieved. During the retrieval of the first answer, there is no influence of spreading activation because the imaginal buffer is empty. The retrieved chunk is placed into the imaginal buffer and becomes $c_{\text{r,imaginal}} = c_r$ where it will influence memory retrieval for the second answer through spreading activation.

Response Mapping

The mapping between the truth value of the retrieved chunk and the response y for person p is given by:

$$y_p = \begin{cases} \text{yes} & c_r(\text{truth}) = \text{true} \\ \text{no} & c_r(\text{truth}) = \text{false} \end{cases}$$

Retrieval Probability

The retrieval probability is found by comparing the response set $W_x \subset R$ to the retrieval set R. The response set is the subset of chunks in the retrieval set that map to the observed response y_p. The response set for yes and no are defined as:

$$W_{\text{yes}} = \{c_m \in R : c_m(\text{truth}) = \text{true}\}$$

$$W_{\text{no}} = \{c_m \in R : c_m(\text{truth}) = \text{false}\}$$

The probability of responding x on the first question is given by the following softmax function:

$$\Pr(Y_p = \mathrm{x}) = \frac{\sum_{\mathrm{c_m \in W_x}} \mathrm{e}^{\frac{\mu_\mathrm{m}}{\sigma}}}{\sum_{\mathrm{c_k \in R}} \mathrm{e}^{\frac{\mu_\mathrm{k}}{\sigma}}} = \frac{\mathrm{e}^{\frac{\beta}{\sigma}} \sum_{\mathrm{c_m \in W_x}} \mathrm{e}^0}{\mathrm{e}^{\frac{\beta}{\sigma}} \sum_{\mathrm{c_k \in R}} \mathrm{e}^0} = \frac{|\mathrm{W_x}|}{|\mathrm{R}|}$$

where μ is mean activation, $\sigma = s\sqrt{2}$ controls activation noise and s is the logistic scale parameter. The expression simplifies to the ratio of chunks leading to response x over all chunks that match the retrieval request because $e^{\beta/\sigma}$ can be factored out of each term. We can rewrite the expression in terms of the number of true and false statements for Clinton and Gore. Let $T = T_c + T_g$ be the total number of chunks containing a true statement and $F = F_c + F_g$ be the total number of chunks containing a false statement, where subscript c represents Clinton and subscript g represents Gore. For example, the probability of responding yes to Clinton on the first question is defined as

$$\Pr(Y_c = \text{yes}) = \frac{T_c}{T_c + F_c}$$

which is simply the ratio true statements made by Clinton compared to all statements made by Clinton. The expression for the second question includes a term for spreading activation, which can be simplified as: $z\left(\frac{1}{x+1}\right)^h$ where $h = \frac{1}{|Q|\sigma}$ and $z = e^{\gamma h}$. For example, the probability of responding yes to Gore on the second question given a response of yes to Clinton on the first question is defined as:

$$\Pr(Y_g = \text{yes} \mid Y_c = \text{yes}) = \frac{T_g \cdot z \cdot \left(\frac{1}{T+1}\right)^h}{T_g \cdot z \cdot \left(\frac{1}{T+1}\right)^h + F_g}$$

In this example, spreading activation increases the probability of responding yes to the question about Gore. The full set of equations can be found in Table 2 for the Clinton-Gore order and Table 3 for the Gore-Clinton order. Note that each joint probability table sums to 1 as required by classical probability theory. Under certain conditions, however, spreading activation causes the probability mass to shift to different cells in each table, producing two different joint probability distributions. In some sense, this is similar to using a different set of basis vectors to define events in the quantum model. In the ACT-R model, the table for each order is consistent with classical probability theory. Similarly, in the quantum model, probabilities based on projection any of a set of orthonormal basis vectors are consistent with classical probability theory. However, just as the ACT-R model is not necessarily consistent with classical probability theory across tables, the quantum model is not necessarily consistent with classical probability theory across different rotations of the basis vectors.

Predictions

In what follows, we describe the predictions of the ACT-R model for order effects and the QQ equality. Although we have proved the following properties, the proofs are omitted due to space limitations.

Table 2: Predictions of the ACT-R order model for the Clinton-Gore order. Column and row labels C and G correspond to Clinton and Gore. Subscripts y and n correspond to yes and no.

	G_y	G_n
C_y	$\frac{T_c}{T_c+F_c} \cdot \frac{T_g \cdot z \cdot \left(\frac{1}{T+1}\right)^h}{T_g \cdot z \cdot \left(\frac{1}{T+1}\right)^h + F_g}$	$\frac{T_c}{T_c+F_c} \cdot \frac{F_g}{T_g \cdot z \cdot \left(\frac{1}{T+1}\right)^h + F_g}$
C_n	$\frac{F_c}{T_c+F_c} \cdot \frac{T_g}{T_g + F_g \cdot z \cdot \left(\frac{1}{F+1}\right)^h}$	$\frac{F_c}{T_c+F_c} \cdot \frac{F_g \cdot z \cdot \left(\frac{1}{F+1}\right)^h}{T_g + F_g \cdot z \cdot \left(\frac{1}{F+1}\right)^h}$

Table 3: Predictions of the ACT-R order model for the Gore-Clinton order. Column and row labels C and G correspond to Clinton and Gore. Subscripts y and n correspond to yes and no.

	G_y	G_n
C_y	$\frac{T_g}{T_g+F_g} \cdot \frac{T_c \cdot z \cdot \left(\frac{1}{T+1}\right)^h}{T_c \cdot z \cdot \left(\frac{1}{T+1}\right)^h + F_c}$	$\frac{F_g}{T_g+F_g} \cdot \frac{T_c}{T_c + F_c \cdot z \cdot \left(\frac{1}{F+1}\right)^h}$
C_n	$\frac{T_g}{T_g+F_g} \cdot \frac{F_c}{T_c \cdot z \cdot \left(\frac{1}{T+1}\right)^h + F_c}$	$\frac{F_g}{T_g+F_g} \cdot \frac{F_c \cdot z \cdot \left(\frac{1}{F+1}\right)^h}{T_c + F_c \cdot z \cdot \left(\frac{1}{F+1}\right)^h}$

Order Effects

According to the model, order effects depend on the ratio of true to false statements for each person. Four order effects can be obtained by subtracting the corresponding cells of Gore-Clinton and Clinton-Gore joint probability tables. Figure 1 shows that the predicted order effect depends on the ratio of true to false statements for each person. For example, in the top left panel, T_c is varied from 0 to 6 while $F_c = T_g = F_g = 1$. The order effect is large when $T_c = 0$ and small to moderate for all other values. Similar patterns can be found in the remaining panels.

Equal Ratios The model predicts no order effects for matching responses (e.g. yes, yes) when the ratios are equal. However, the order effect for a yes, no response is the negative of the order effect for a no, yes response, which ranges over negative and positive values. More formally, we define O_{y_c,y_g} as the order effect for response y_c to the Clinton question and y_g to the Gore question. With this notation, we can write: $O_{\text{yes,yes}} = O_{\text{no,no}} = 0$ and $O_{\text{yes,no}} = -O_{\text{no,yes}}$ if $T_c = v \cdot T_g$, $F_c = v \cdot F_g$. As a special case, if $T_c = F_c$ and $T_g = F_g$, then $O_{\text{yes,no}} = O_{\text{no,yes}} = 0$. Intuitively, this means that the effects of spreading activation in both question orders cancel out because the ratios are 50-50, thus eliminating the order effect. This can be seen in Figure 1 where the values on the x-axis equal 1.

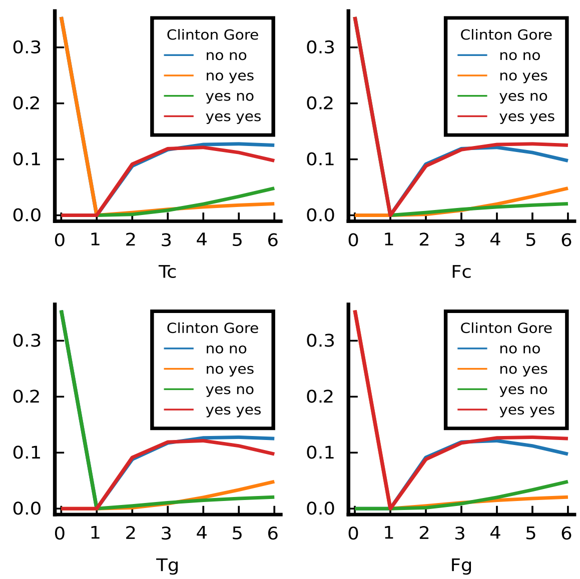

Figure 1: Order effect predictions. Each plot shows the absolute order effect along the y-axis for the number of chunks of the type specified in the x-axis while the chunk types are hold constant at a value of 1.

QQ Equality

The ACT-R model follows the basic mathematical constraint whereby $q_1 = -q_2$, but satisfies the QQ equality ($q_1 = q_2 = 0$) under specific conditions that depend on the ratio of true to false statements for each person. Figure 2 shows the q values as a function of the value on the x-axis while the other three values are fixed at 1. When value on the x-axis is zero, a large violation of the QQ equality is predicted. In other cases, the predicted violation of the QQ equality is small or zero.

Equal Ratios ACT-R satisfies the QQ equality when the ratios of true to false statements are equal for each person. More formally, $q_1 = q_2 = 0$ if $T_c = v \cdot T_g$, and $F_c = v \cdot F_g$. This occurs in Figure 2 where the value on the x-axis is equal to 1.

Complementary Ratios ACT-R satisfies the QQ equality when the ratios of true to false statements are complementary for each person, but the number of chunks for each person are equal. More formally, $q_1 = q_2 = 0$ if $T_c = F_g$ and $T_g = F_c$.

Q Distribution To investigate the extent to which the ACT-R model violates the QQ equality, we computed q values across all permutations of memory composition for values from 0 to 6. As shown in Figure 3, most of the density is centered near zero. In particular, approximately 77% of q values range between -.1 and .1 and approximately 68% range between -.05 and .05. The remaining portion of the distribution extends towards -.4 and .4 in a roughly uniform manner. Collectively, these results suggest that violations of the QQ equality are typically small, but can be quite large under some circumstances.

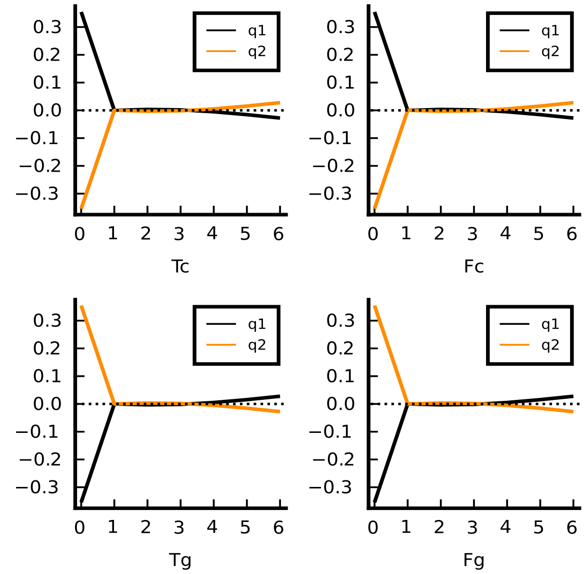

Figure 2: q-value predictions. Each plot shows the q values for the number of chunks of the type specified in the x-axis while the other chunk types are hold constant at a value of 1.

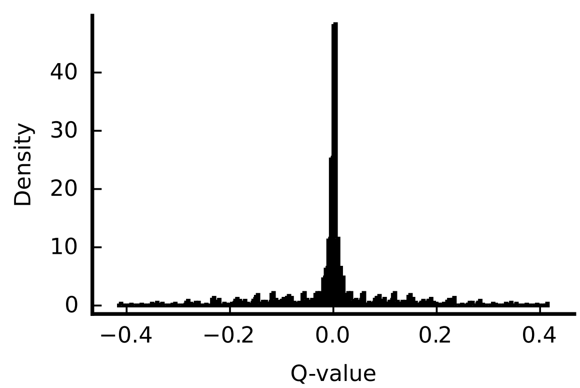

Figure 3: Distribution of q values across all $7^4 = 2,401$ permutations of T_c, F_c, T_g, and F_g for values 0 to 6.

Discussion

In the present paper, we developed a memory-based model of order effects within the ACT-R cognitive architecture and outlined many of its predictions and properties. Our analysis reveals that ACT-R can produce order effects and satisfy the QQ equality depending on the composition of declarative memory. Across a large range of memory sets, the model produces q values that either satisfy the QQ equality or violate it only by a small degree. In other cases, there is a clear divergence from the QQ equality.

Some points of similarity and difference between the quantum model and the ACT-R model are worth noting. One point of similarity is that context is an important determinant of order effects in both models. In the quantum model, order effects arise from non-commutative evaluation processes when events are incompatible. In the ACT-R model, the answer to the first question creates a new context through which spreading activation modulates the retrieval probabilities for the second answer. The models differ in several important regards. One difference is the distinction between memory-based vs. online judgments (Hastie & Park, 1986). In ACT-R, judgments are formed from a set of experienced events stored in memory, whereas in the quantum model, judgments are constructed online through comparison processes and do not require a definite reference class. One direction for future research is to determine whether ACT-R can perform online judgments by comparing chunks in different buffers.

Although we have demonstrated as a proof of concept that ACT-R can produce order effects and satisfy the QQ equality under specific conditions, the model has not been tested against empirical data. For this reason, it remains unclear how it compares to the quantum model in terms of empirical support. As a memory-based model, ACT-R requires a well-controlled experiment in which the composition of declarative memory is manipulated to test the properties outlined above. Existing data sets are not suitable for testing the ACT-R model because factors influencing memory were not controlled or measured. For example, respondents in the national survey likely differed in terms of political knowledge and information sources, which, in turn, could introduce heterogeneity in judgments about Clinton and Gore. Uncertainty and heterogeneity in memory composition would render the results uninterpretable from the standpoint of the ACT-R model. In future research, we plan to design a memory based experiment to test the predictions outlined above.

Our analysis shows that the predictions hold for different values of maximum associative strength and activation noise so long as maximum association strength is sufficiently large to produce a positive spreading activation term. One may wonder how the predictions might change when certain assumptions of the model are relaxed. For example, relaxing the assumption that β is equal across chunks leads to a somewhat more complex model, but the predictions ultimately depend on the ratio of activation of true and false statements for each person rather than the ratio of chunks.

Conclusion

Order effects are an interesting benchmark for testing the ACT-R cognitive architecture because it was not developed to account for such effects. Nonetheless, we demonstrated that ACT-R can produce order effects using existing memory retrieval mechanisms and can satisfy the QQ equality under some conditions. Although more work is required to test the model, we regard this proof of concept as an important first step towards stress testing the architecture against new benchmark phenomena.

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