A quantum walk framework for multialternative decision making

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Introduction

Traditionally, evidence accumulation in multialternative decision making is modeled by the Markov random walk process (**MRW**) (Roe, Busemeyer, & Townsend, 2001; Usher & McClelland, 2001; Bhatia, 2013; Noguchi & Stewart, 2018, 2018). Despite their all-around successes, these MRW models are challenged by recent evidence of Markov violations in evidence accumulation including interference effects of choice on confidence for multistage decision making (Kvam, Pleskac, Yu, & Busemeyer, 2015), interference effects of confidence on confidence (Busemeyer, Kvam, & Pleskac, 2020), and order effects in experimental test of attraction effects (Trueblood & Dasari, 2017).

On the other hand, the quantum walk process (**QW**) (Busemeyer, Wang, & Townsend, 2006; Wang, Solloway, Shiffrin, & Busemeyer, 2014) explain these Markov violations in a natural way. However, QW models have only been applied to binary alternative decision making, and this raises the questions of whether we can extend existing QW models to explain both Markov violations and traditional context effects in multialternative decision making. Our goal here is to present a general framework for this potential extension.

Quantum walk model for binary alternative

Quantum walk (**QW**) is the quantum analogy of Markov random walk (**MRW**) which, instead of describing the time evolution of an initial probability distribution, describes that of an initial probability amplitude distribution. The quantum time evolution is governed by Schrödinger's equation:

$$\frac{d}{dt}\psi(x,t) = -i \cdot H \cdot \psi(x,t), \tag{1}$$

where ψ is the probability amplitude distribution (quantum wavefunction), and H is the Hamiltonian operator in analogous with the Markov transition rate matrix. For discrete-finite-state quantum walk, H can be written in the following $N \times N$ matrix form:

$$\begin{cases} H(i,i) = u(i), \text{ for } 1 \le i \le N \\ H(i+1,i) = H(i,i+1) = \sigma^2, \text{ for } 1 \le i \le N-1, \end{cases} \tag{2}$$

where $u(x)$ denotes the potential function, and σ^2 is the diffusion rate that describes the effect of a constant non-conservative force acting on the system. The solution to Schrödinger's equation gives:

$$\psi(x,t) = U^t \cdot \psi(x,0) = e^{-i \cdot H \cdot t}\psi(x,0), \tag{3}$$

where $U = e^{-i \cdot H}$ denotes the quantum unitary operator. In binary alternative decision making problem to which QW is previously applied (Busemeyer et al., 2006), $\psi(x,t)$ can be viewed as a probability amplitude distribution over the confidence states. $u(x) = \alpha x + \beta$ is modeled by a linear function with drift parameter α.

Multi-alternative quantum walk framework

The multi-alternative quantum walk model (**MQW**) is inspired by the existing QW model for binary choice decision making and MRW models that use multiple accumulators to explain multialternative decision making. MQW is defined by (1) initial state, (2) Hamiltonian that describes how the initial state evolves (3) stopping conditions.

Initial state Suppose there are $N \ge 3$ alternatives to choose from, we define N initial states with $\psi(x,0)_m$ being the state for the mth alternative. The aggregated initial state is written as a direct sum:

$$\psi(x,0) = \bigoplus_{m=1}^{N} w_m \cdot \psi(x,0)_m, \tag{4}$$

where w_m with $\sum_{m=1}^{N} |w_m|^2 = 1$ models the attention weights to each alternative. By the definition of direct sum, if each $\psi(x,0)_m$ is of dimension $P \times 1$, then $\psi(x,0)$ will have dimension $NP \times 1$.

Hamiltionian To describe the time evolution, for each of the alternatives denoted as A_m, we define N Hamiltonian matrices, where the qth of such denoted as $H_{m,q}$ has dimension $P \times P$. This $H_{m,q}$ represents how evidence accumulation of alternative A_m influences evidence accumulation of alternative A_q, and thus can be used to model context effects in multialternative decision making. For example, in the case of similarity effects, accumulating evidence in favor of A_m inhibits accumulating evidence in favor of the similar alternative A_q, and thus time evolution described by $H_{m,q}$ will decrease A_q's confidence rating. In cases when A_m and A_q are independent, $U_{m,q} = e^{-i \cdot H_{m,q}}$ will be the identity matrix, and $H_{m,q}$ will thus be the zero matrix.

According to equation 2, we need to define a potential function $u_{m,q}(x)$ and a diffusion rate $\sigma_{m,q}$ for each $H_{m,q}$. Similar to Busemeyer et al. (2006), we make $\sigma_{m,q}$ a free parameter, and $u_{m,m}(x) = \alpha_{m,m}x + \beta_{m,m}$ a linear potential function with free parameters $\alpha_{m,q}$ and $\beta_{m,q}$. To further constrain the number of parameters, we make $\sigma_{m,q} = \sigma_q$ for each alternative A_q, which means that the diffusion effect on A_q is independent of A_m. In the most general case, there will be a