Estimating ACT-R Declarative Memory Parameters Using a Drift Diffusion Model

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Abstract

Accurately fitting cognitive models to empirical datasets requires a robust parameter estimation process which is often arduous and computationally expensive. A way to mitigate this challenge is to integrate participant-specific and efficient mathematical models such as a drift diffusion model (DDM) into the parameter estimation process of cognitive modeling. In this study, we exhibit a clear mapping of the parameters outputted by DDM onto the declarative memory parameters utilized in the cognitive architecture, ACT-R. We show a fairly consistent recovery of simulated ACT-R parameters using DDM and a successful application in using this method to optimize ACT-R simulated fit to an empirical dataset. Notably, we show that the DDM-derived estimated parameters are individualized to the original participant, providing a unique opportunity for parsing out individual differences in cognitive modeling. This method outlined here allows one to estimate ACT-R parameters without the need to manually build and run an ACT-R model while also allowing for neural contextualization of DDM parameters.

Keywords: Drift Diffusion Model, Cognitive Architecture, Computational models, Individual Differences

Introduction

A common challenge associated with cognitive modeling is how to accurately capture individual differences within the parameters that comprise these models. Parameter estimation now relies on unfastidious and computationally expensive methods such as manual parameter grid-searches. Incorporating a statistically rigorous and behaviorally-valid computational model such as a drift diffusion model into the parameter estimation process of ACT-R may allow for better empirically-informed ACT-R models. Similarly, although DDM has been widely replicated in behavioral paradigms and the outputted parameters show distinct and replicable behavioral correlates (Ratcliff & Tuerlinckx, 2002; Voss et al., 2004), studies examining the neural substrates of the DDM parameters have large variability in their results (Gupta et al., 2022). Integrating DDM into a well-established cognitive architecture such as ACT-R (Anderson, 2007) would allow DDM parameters to have robust neural correlate interpretations. Further, accurate ACT-R parameter estimation would eliminate the need for the modeler to manually build and run an ACT-R model to use for neural or cognitive interpretation in the context of declarative memory tasks, increasing the accessibility of these methods to a wider array of non-modeler researchers.

ACT-R Declarative Memory

ACT-R is a well-established cognitive architecture that includes a highly reliable model of declarative memory (Anderson et al., 2004; Anderson, 1974; Kotseruba & Tsotsos, 2020; Pavlik & Anderson, 2005). Declarative memories or knowledge within ACT-R are encoded in record-like structures called chunks, representing semantic memories. ACT-R's declarative memory module functions by making less used chunks harder to retrieve over time through their assigned activations. Chunks are selected on the bases of their *activation*, a quantity that reflects the log odds that the chunk will be needed. Specifically, the activation A_c of a chunk c at time t is computed as:

$$A_c = \sum_i (t_i - t)^{-d} \quad (1)$$

where t_i represents the time of the i-th event in which c was encoded or retrieved. Retrieval of information from memory can be viewed as a process of evidence accumulation, where environmental or internal cues contribute evidence to competing chunks within one's memory. These chunks are competing for retrieval and the first chunk to accumulate enough evidence to be chosen, crosses a "decisional threshold" and a response is initiated (Anderson, 2007).

Drift Diffusion Model

A drift diffusion model (DDM; Ratcliff, 1978; Voss et al., 2013) has been proposed to model a two-alternative forced-choice task and is based on early models of the continuous random walk process (Stone, 1960; Wald & Wolfowitz, 1948). The DDM is based on several basic assumptions: during a binary decision process, information will accumulate at a continuous rate and this accumulation process can be explained using a Weiner diffusion process (Ratcliff & McKoon, 2008; Ratcliff & Tuerlinckx, 2002). Information accumulation is characterized by a constant systemic component with an added component of normally distributed random noise. This assumption of random noise is meant to emulate repeated processing of the same stimulus or same type of stimulus and explains the variance in response times and erroneous response errors observed in empirical reaction/accuracy distributions (Ratcliff & McKoon, 2008; Ratcliff & Tuerlinckx, 2002; Voss et al., 2013). The decision process is terminated as soon as the systemic counter accumulates information to the point of reaching one of the two decisional thresholds. The basic model can be depicted in Figure 1A.

A drift diffusion model is distinguished by its distinct parameters estimated from empirical decision time distributions. The first parameter, or drift rate (v) is calculated through the average of the rate of evidence accumulation from the start of the decision process (beginning of evidence accumulation) until a decision is made (evidence accumulator reaches either upper or lower decisional threshold). Previous studies have shown that drift rate can be interpreted as a measure of cognitive speed and is affected by value associated with the stimulus as well as the separation between choices (Bond et al., 2018; Ratcliff & Frank, 2012). We are similarly able to estimate the decisional threshold (a). The decisional threshold represents the amount of evidence needed to make a decision. A higher decisional threshold indicates a larger distance between the lower and upper decisional thresholds. Decisional threshold has been shown to highly depend on a speed-accuracy tradeoff and is sensitive to changes in instructions emphasizing speed over accuracy or vice versa (Mulder et al., 2013). We are also able to calculate the decisional starting point, or decisional bias (z). The decision starting point represents the starting bias at the beginning of the decision process and represents the relative distance to the upper/lower decisional threshold. A higher decision starting point would represent bias towards the upper decisional threshold. Finally, we are able to estimate the extradecisional time component (t_0) which represents the time used to complete all processes not directly related to the decisional process such as stimulus encoding or motor execution of the response.

Mapping DDM Parameters onto ACT-R

Recent work has shown that we can treat the ACT-R declarative memory module as an evidence accumulator model, and therefore can map the actual evidence accumulator model (DDM) parameters onto the declarative memory parameters within ACT-R (van der Velde et al., 2021). The total time required to retrieve the winning chunk c with activation A_c within ACT-R is defined by the equation below. Included in the equation is the latency factor F.

$$RT_c = Fe^{-A_c} + t_{er} \qquad (2)$$

Over a trial average, this equation can be rewritten to derive the expected time for retrieval across a series of trials using the average latency factor $\bar{F}$ and average activation A_c:

$$E(RT_c) = \frac{\bar{F}}{e^{A_c}} + t_{er} \qquad (3)$$

As the DDM assumes evidence accumulation at a constant rate, the expected time for accumulator c to reach the upper decisional threshold a is dependent on the decisional starting point z and drift rate v with a scaling factor t_0 (Bogacz et al., 2006).

$$E(RT_c) = \frac{a-z}{v} + t_0 \qquad (4)$$

The DDM is different from other evidence accumulator models in which there are two separate accumulation processes occurring for each choice (or chunk) as DDM incorporates the difference of the two possible decisions into the evidence accumulation process (Bogacz et al., 2006). In DDM, the probability P of accumulator c with drift rate v of reaching the upper decisional threshold a is defined by the equation below.

$$P_c = \frac{1}{1+e^{-2v}} \qquad (5)$$

This equation is reminiscent of the probability of receiving a certain chunk over a competitor in ACT-R: The probability P of retrieving chunk c with activation A_c over a foil f with activation A_f can be represented by the equation below.

$$P_c = \frac{e^{A_c}}{e^{A_c}+e^{A_f}} = \frac{1}{1+e^{A_c-A_f}} \qquad (6)$$

Using the above equations, we can then map ACT-R parameters onto those outputted by DDM (Figure 1B). $\bar{F}$ in ACT-R (latency factor) is related to the relationship between the upper decisional threshold and the decisional starting point or bias in DDM.

$$\bar{F} = a - z \qquad (7)$$

Drift rate v within DDM is related to the difference between the activations of the competing chunks within ACT-R. Here, we adapted the equation to reflect the difference of the average activations of competing chunks c and f represented by ΔA.

$$\Delta A = -2v \qquad (8)$$

Similar to previous work, we see a direct equivalency of the extradecisional component within an evidence accumulator model of DDM and that within ACT-R (van der Velde et al., 2021).

$$T_{er} = t_0 \qquad (9)$$

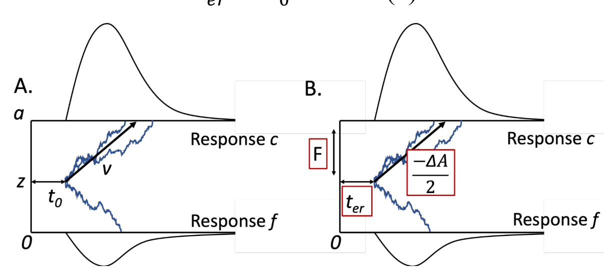

Figure 1: (A) Illustration of the diffusion model with the four main parameters (a, z, v, and t_0) with three exemplary trials (in blue). (B) The same model depiction but with the equivalency of ACT-R parameters using equations (7)-(9).

Simulation: Recovering ACT-R Parameters

Materials and Methods

Data. The data used in this analysis was simulated using ACT-R with code adapted from van der Velde et al. 2021. ACT-R was used to simulate 25 model participants undergoing a declarative memory retrieval task with two competing chunks, *c* and *f*. Time to reach the decision boundary of the winning chunk was recorded in seconds (referred to as response time). A "correct" trial was indicated when chunk *c* was the first accumulator to reach the decision boundary. Overall DDM fit can be affected by outlier reaction times (Lerche et al., 2017) at lower trial numbers so an IQR outlier correction was applied to simulated data prior to model fitting. Simulations were repeated with a varying number of trials per model participant, ranging from 25 to 5000 to best understand the minimum trial size needed for accurate parameter recovery.

Model Fitting. The DDM was fitted individually to each model's simulated response/accuracy distributions using the *ddiffusion* density function within the *rtdists* package in R (R version 3.2.0; *rtdists* 0.8-3). For each model participant, we used DDM to estimate parameters a, z, v, and t_0. We likewise allowed the model to fluctuate on an inter-trial basis by including inter-trial variability parameters that account for changes in t_0, z, and v from trial-to-trial (variability parameters: st_0, sz, sv). These parameters have been shown to help with DDM fit to the empirical distribution and improve accuracy of parameter estimation (Lerche & Voss, 2016). ACT-R parameters (ΔA, $\bar{F}$, T_{er}) were recalculated using the equations (7)-(9) previously described.

Results

To understand the optimal trial size for consistent ACT-R parameter recovery we attempted the parameter recovery simulation at varying trial sizes from 25 trials per participant to 5000. Across all trial sizes we calculated absolute error and correlations across the recovered parameters: $\bar{F}$, T_{er} and ΔA. Notably, we saw comparable absolute errors and correlations of original vs. recovered parameters at trial sizes of 100 trials per simulated participant or greater (Figure 2; T_{er} at 100 trials per participant: $r = 0.97$, T_{er} at 5000 trials per participant: r = 0.99; $\bar{F}$ at 100 trials per participant: r = 0.46, $\bar{F}$ at 5000 trials per participant: r = 0.48).

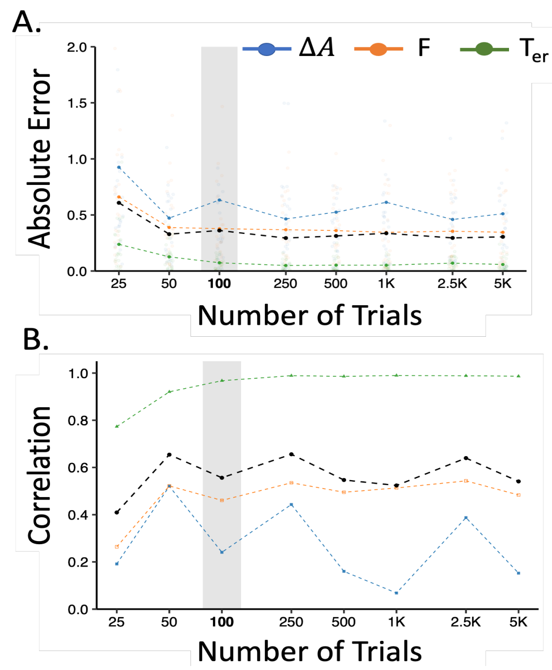

Figure 2: Absolute error (A) and Pearson correlation values (B) across trial sizes 25-5000 per participant across the three recovered parameters: ΔA (shown in blue), $\bar{F}$ (shown in orange), and T_{er} (shown in green). Mean absolute error (A) or mean correlation (B) across all parameters is shown in black.

With just 100 trials per participant, the original inputted ACT-R parameters showed a fairly linear and consistent recovery with DDM parameter estimation (Figure 3).

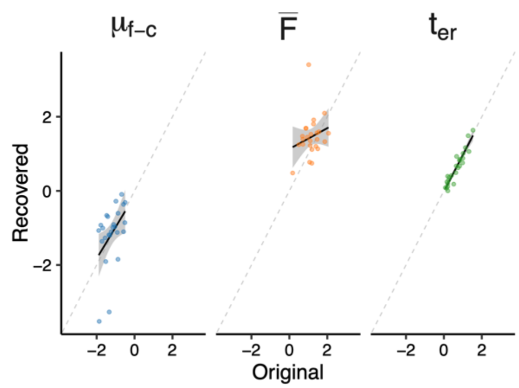

Figure 3: Scatter plot of original (x-axis) versus recovered (y-axis) parameter values for 25 model participants with 100 trials per participant for the three recovered parameters: ΔA (left: shown in blue), $\bar{F}$ (center: shown in orange), and T_{er} (right: shown in green).

We do, however, see larger variability in the recovery of the difference of activation rates (ΔA) with few outlier participants causing large increases in the error observed in the recovery. This effect did not seem to reduce with increased trials per participant (Figure 4).

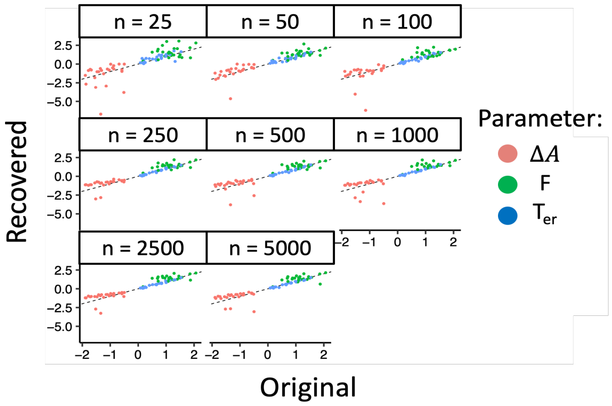

Figure 4: Scatter plot of original (x-axis) versus recovered (y-axis) parameter values for 25 model participants with 25-5000 trials per participant for the three recovered parameters: ΔA (shown in pink), $\bar{F}$ (shown in green), and T_{er} (shown in blue).

Parameter Estimation in an Empirical Dataset

Materials and Methods

Data. The data used here come from an experiment carried out by Verstynen (2014) and freely available on OpenNeuro (dataset ds000164). Twenty male and ten female participants performed the color-word Stroop task (Botvinick et al., 2001; Gratton et al., 1992; MacLeod, 1991; Stroop, 1935) which consisted of congruent, incongruent, and neutral stimulus conditions. Participants were presented with word-stimuli and were instructed to respond to the color in which the word was printed and to ignore the meaning of the printed word. In a congruent condition, the words "GREEN", "BLUE", and "YELLOW" were displayed in the colors green, blue, and yellow respectively. The incongruent condition showed words whose meaning was a different color than the ink in which the printed word was displayed (i.e., the displayed word was "GREEN" in blue ink). In neutral conditions, a non-color word was presented in an ink color (i.e., the word "HAT" printed in blue ink). Participants responded by pressing different buttons, with different right-hand fingers, for each color (e.g., red: index; green: middle; and yellow: ring finger). Each participant completed 120 trials (42 congruent, 42 neutral, 36 incongruent). Trial types and stimuli types were pseudorandomized in an event-related fashion. Response time and accuracy were recorded for each trial. The data was collected as part of a larger study and more information of the participants and procedure can be found in Verstynen (2014).

Model Fitting. The DDM was fitted to each participant's response-accuracy distribution separately. To optimize computing speed and for added statistical rigor, DDM was fitted using the *Fast-dm-30.2* toolbox (Voss & Voss, 2007). Each participant's parameter optimization was statistically verified using the Kolmogorov-Smirnov method. Similar to the simulation experiments, inter-trial variability parameters (st_0, sz, sv) were allowed to fluctuate across trials during the parameter estimation process to optimize DDM fit. ACT-R parameters ($\Delta A, \bar{F}$, T_{er}) and were again recalculated from the outputted DDM parameters (v, z, a, and t_0) using equations (7)-(9). DDM density plots were created using the *ddiffusion* density function within the *rtdists* toolbox in R (R version 3.2.0; *rtdists* 0.8-3).

ACT-R Stroop Task. A simple model of the Stroop task was implemented to test the possibility of translating DDM parameters directly into ACT-R models. This model borrows the central idea of previous models of response interference in the Stroop (Lovett, 2002) and Simon tasks (Stocco et al., 2017) and captures the Stroop effect as interference in the color name retrieval due to competing sources of activation. Specifically, the model responds by initially focusing on the word's color. While attending to the color, the model attempts to retrieve an associated color name. This retrieval process is aided by activation spreading from the attended color to the corresponding name (e.g., from the color green to the word "green"), which confers an additional boost of activation to the correct color name over the equally active names of other colors. Once a color name is retrieved, a production rule performs the corresponding motor response. The simplicity of this model makes the DDM parameters immediately translatable. Specifically, the difference in mean activation between competing chunks (ΔA) corresponds to the contribution of spreading activation from the word's color, and the T_{er} parameter corresponds to the duration of motor execution (the "motor burst time" parameter) once the visual encoding time (fixed and maintained at its default value of 50ms) and the execution time of the necessary productions (three productions for 50ms each) are accounted for.

Note that although T_{er} by definition represents time components split across both the visual encoding and motor module, functionally it does not make a difference which of these T_{er} is incorporated into as regardless it will be added on to overall reaction time. We ran individualized ACT-R models with these inputted parameters for each of the participants with the same number of trials as in the empirical study (42 congruent, 42 neutral, and 36 incongruent).

Results

We fit DDM to each participant's data individually. Across all participants, we observed a reasonable fit of DDM to the empirical distribution which was further verified through the Kolmogorov-Smirnov test statistic (p = [0.83–0.99] across all stimulus types). This provided reassurance that the outputted DDM parameters were reasonably estimated and could be used for subsequent ACT-R parameter recovery. Excitingly, we were able to estimate reasonable ACT-R parameters: $\bar{F}$, T_{er}, and difference of activation rates between the competing chunks (ΔA). Although we observed moderate variability across subjects and condition types, $\bar{F}$ (across all conditions $\bar{F} = 0.64 \pm 0.13$), T_{er} (across all conditions T_{er} = 0.61 ± 0.07), ΔA (across all conditions $\Delta A = 6.13 \pm 1.74$) and were within typical ranges according to previous ACT-R studies (Anderson et. al, 1998).

We were further interested to see if ACT-R simulated data of a Stroop task that utilizes these estimated parameters would provide a comparable reaction time/accuracy distribution to the empirical data we originally inputted into the DDM. Across the 28 participants we saw relatively linear recovery of mean reaction time and accuracy across participants (Figure 6A). To ensure these parameters were indeed individualized to the participant and not a factor of task, we randomized the estimated parameters across participants and again compared the recovery of mean reaction time/accuracy across participants. As seen in Figure 6B, this recovery is substantially worse if parameters are not matched to the original participant, providing evidence that this parameter estimation method is sensitive to and sustains individual differences in its integration into ACT-R.

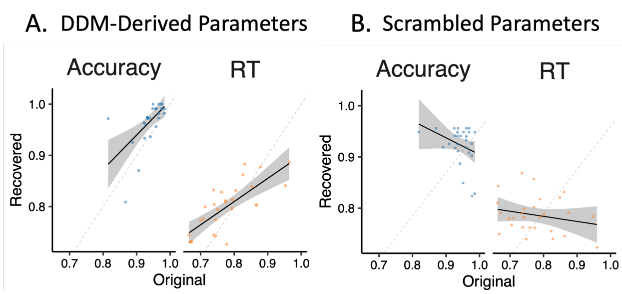

Figure 6: Mean accuracy and reaction time of the original empirical subjects (x-axis) versus the ACT-R simulated data (y-axis) with the DDM-Derived participant-specific ACT-R parameters inputted (A) versus if the DDM-Derived ACT-R parameters are randomized across different subjects (B).

Discussion

In this paper, we have presented evidence of an ability to integrate DDM parameters into the ACT-R parameter estimation process. Across trial sizes as low as 100 trials per model participant, we observed a fairly consistent and linear recovery of the extradecisional time component T_{er}, the latency factor $\bar{F}$, and difference of activation rates between the top two competing chunks ΔA, within a simulated declarative memory retrieval task. Both T_{er} and $\bar{F}$ showed a relatively consistent increase in correlation and decrease in observed absolute error as trial sizes increased from 25-5000 trials per participant. Interestingly, in observing the recovery of ΔA, we observed a "zig-zag" pattern in correlation and observed absolute error as trial sizes increased instead of the steady increase in recovery correlation/decrease in absolute error as observed with the other parameters. We expect this is due to the presence of 1-4 simulated participants within each simulation in which the estimated DDM drift rate (v) was very high due to the presence of numerous trials with extremely short simulated reaction times (<200ms). As our simulated reaction time/accuracy distributions were drawn from random distributions, the presence of model participants with trials like this were randomly observed, which caused the odd pattern of recovery (i.e., seemingly better observed absolute error in trial sizes of 50 compared to 100 trials per participant). In use with empirical data and non-simulated participants, this becomes less of an issue as extremely short reaction times are typically removed by way of outlier correction prior to model fitting. However, to not only reduce the presence of these apparent outliers but similarly increase the statistical rigor of the DDM parameter estimation, we plan to integrate an optimizer function into the process of fitting the DDM to the original dataset. From there, one could choose the set of parameters with an optimized fit before mapping to ACT-R parameters. One could similarly utilize existing software such as the *Fast-dm-30.2* toolbox (Voss & Voss, 2007) which incorporates optimization methods without added burden to the user.

To further emulate this method's applicability, we utilized this DDM-ACT-R parameter estimation method on an empirical data set of a Stroop task (Verstynen, 2014). We demonstrated that by using DDM-derived parameters, we were able to estimate ACT-R parameters within typical ranges according to prior studies. Most excitingly, when we integrated these DDM-derived parameters into an ACT-R simulated Stroop model, we were able to accurately recreate the reaction time/accuracy distributions observed within the empirical dataset as shown by comparing empirical versus recovered mean reaction time and accuracies. Notably, these parameters seemed to be individualized to the participant, as randomization of these parameters showed a worse recovery of empirical mean reaction time and accuracy across participants. Further comparison experiments are needed to understand whether DDM-ACT-R parameter estimation method is indeed more accurate/individualized compared to common parameter estimation methods such as parameter grid searches or sweeps, although this DDM-ACT-R method has been shown to be quicker and less computationally expensive in this application.

While this method has shown promising results in optimizing incorporating empirical data into a simulated model, the Stroop ACT-R model we used is significantly simplified compared to existing models of this task that have been based on the neurocognitive properties this task elicits (Lovett, 2002; Stocco et al., 2017). In applications confined to a declarative memory task, we are hopeful that this method will be relevant beyond binary decision tasks to multi-alternative decisions, again increasing the usability of drift diffusion models. However, future work utilizing this method outside of the scope of a declarative memory task (i.e., one that relies on procedural complexity) is needed to understand the breadth of its applicability.

Individualized, consistent and accurate estimation of ACT-R parameters with this method, even on simple tasks, would allow us to have a proxy measure for task neural dynamics in datasets that only have behavioral data, greatly reducing the need for expensive and time-consuming fMRI data collection. The integration of DDM into ACT-R can further give neural context to the parameters used in DDM, an application of DDM that has been inconsistent in previous work (Gupta et al., 2022).

In summary, we have exhibited a clear integration of the drift diffusion model into the cognitive architecture of ACT-R. This relationship contributes to a larger effort in optimizing the utilization of empirical data in informing cognitive models as well as in the overall integration of modeling methods.

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